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Nonlinear Dynamics and Chaos: With Applications to Physics, Biology, Chemistry, and Engineering. *By Steven H. Strogatz.* Addison-Wesley, Reading, Massachusetts, 1994, 498 pp. $55.95, hardcover, ISBN 0-201-54344-3.

This book was written as a textbook for an introductory course on nonlinear dynamics at the junior-senior level. It would also serve well as a starting point for a scientist or engineer wanting to learn about the subject on his own. Mathematical prerequisites are minimal: single variable calculus and a course in ordinary differential equations would certainly be sufficient. More advanced ideas are used here and there, but the independent learner would have no trouble with them, and in the classroom setting a few words by the instructor should be adequate.

The text begins with a preliminary chapter of just eleven pages that serves as an introduction to the book. This chapter reviews briefly the historical development of the study of nonlinear dynamical systems and establishes the logical framework in which the topics covered in the book are to be viewed. The rest of the book is subdivided into the following main parts and chapters. Part I, One-dimensional flows (114 pages) 1. Flows on the Line, 2. Bifurcations, 3. Flows on the Circle. This part introduces the geometric point of view on ordinary differential equations and covers, among other things, stability of fixed points, the saddlenode, transcritical, and pitchfork bifurcations, considerations of existence and uniqueness of solutions, and numerical techniques. Part II, Two-dimensional flows (174 pages) 1. Linear systems, 2. Phase plane, 3. Limit cycles, 4. Bifurcations revisited. Aside from the obvious, topics touched on here include index theory and the Poincaré–Bendixson Theorem, conservative systems, Lyapunov functions, relaxation oscillations, and various bifurcations of limit cycles. Part III, Chaos (152 pages) 1. Lorenz equations, 2. One-dimensional maps, 3. Fractals, 4. Strange attractors. Most of the time here is devoted to the Lorenz equations and one-dimensional maps. The "baker's map," Hénon system, and Rössler system are discussed in the last chapter.

The approach throughout is intuitive rather than rigorous. Numbered theorems are rare, and formal proofs are almost entirely absent. Sometimes sketches of the proof of a result is given; oftentimes the reader is referred to other texts or original works, all listed in the bibliography, which is extensive. The point of view is to build understanding and insight through carefully developed discussions and the study of examples, rather than to sift through detailed arguments. The effect is to keep the book fast-paced and interesting. This is not to say that the book is sloppy or inaccurate. On the contrary, it has been carefully and accurately written. I may have detected one misstatement (depending on how it is interpreted) in the entire 450 pages, and that was in an exercise.

This book has three particularly strong features. First, it is exceptionally well-written. Time after time Professor Strogatz gives explanations of concepts that are among the most lucid I have ever read. His examples are clear, the language usage is well above average, and the logical development is natural, smoothly moving from one idea to the next. Moreover, every idea, even coordinate changes, is well-motivated. Second, the text excels in the number and variety of its applications. All the usual examples are there (pendula, populations, balls in rotating hoops, chemical reactions, ...), but one also finds discussions and problems built around such dissimilar phenomena as magnets, lasers, firefly synchronization, Josephson junctions, and biochemical switches. Additionally, the applications of the mathematics are not merely tacked on, but form an integral part of the text. The third strong feature of the book as a textbook is the problems at the end of the chapters. There are plenty of them, some of them straightforward drill problems, some of them the working out of cases of discussions left to the reader, some of them breaking new ground. Some are easy, some much more challenging. Many are based on applications that vary widely. As an aid to the instructor, problems are grouped at the end of each chapter, and are numbered according to the section of the text to which they relate. Answers are provided for selected exercises.

In conclusion, I regard this text as one of the best introductions to nonlinear dynamics currently available. I highly recommend it.

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