Delayed switching in a phase-slip model of charge-density-wave transport

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We analyze the dynamics of the depinning transition in a many-body model of charge-density-wave (CDW) transport in switching samples. The model consists of driven massless phases with random pinning and a coupling term that is periodic in the phase difference, thus allowing phase slip. When the applied field in our model exceeds the depinning threshold by a small amount, there is a delay before the appearance of a coherent moving solution. This delay is also seen experimentally in switching CDW materials. We find that close to threshold the switching delay is approximately proportional to the inverse distance above threshold. Analytical results agree with numerical integration of the model equations. Results are also compared to available experimental data on delayed switching.

I. INTRODUCTION

The nonlinear conduction in charge-density-wave (CDW) materials has been extensively studied in a variety of quasi-one-dimensional metals and semiconductors, and a large number of theoretical models have been presented, each explaining some of the phenomena observed in these materials.[1,2] Classical models of CDW dynamics[3–6] which consider only the phase degrees of freedom of the CDW condensate have been quite successful at describing the behavior of both pinned and sliding CDW's in a regime where pinning forces are weak, phase distortions are small, and higher-energy amplitude modes are not excited. While many aspects of CDW dynamics are well described by a rigid phase model with only one degree of freedom,[6] dynamics near the depinning threshold are best treated by a model with many degrees of freedom, which allows for local distortion of the CDW in response to random pinning forces. For example, a mean-field discrete-site model of many coupled phases analyzed by Fisher[5] gives a continuous depinning transition with a concave-upward *I-V* curve at depinning, in agreement with experimental data, and in contrast to depinning of the corresponding single-phase model.

Recently, experimental and theoretical interest has turned to a class of CDW systems which have discontinuous depinning transitions and hysteresis between the pinned and sliding states.[7–23] This discontinuous depinning has been termed "switching." Several authors[11–17] (though not all[18–21]) have attributed switching to phase slip between coherent CDW domains occurring at ultrastrong pinning sites. Experimentally, it is known that switching can be induced by irradiating the sample, which creates strong pinning sites.

A very interesting phenomenon associated with switching, first reported by Zettl and Grüner[7] for switching samples of $NbSe_3$, and subsequently seen in other materials,[8–10] is a delayed onset of CDW conduction near threshold. When an electric field slightly larger than the depinning threshold is applied to a switching sample, there is a time delay before the pinned CDW begins to slide. The delay grows as threshold is approached from above and varies over several orders of magnitude from tenths to hundreds of microseconds.

In this paper, we analyze the dynamics of depinning for a many-body model of CDW transport in which phase slip is allowed. We show that, for an applied driving field slightly above the depinning threshold, switching from the pinned state to the coherent moving state is delayed, and that the delay grows roughly as the inverse of the distance above threshold. In Sec. II the phase-slip model is described and briefly compared to other models of CDW transport. The dynamics of the depinning transition in the model are then analyzed and an expression for the switching delay is derived. These results are shown to agree with direct numerical integration of the model. In Sec. III our results are compared with the available experimental data on switching delay.

II. PHASE-SLIP MODEL OF DELAYED SWITCHING

A. Mean-field model of switching CDW's

The dynamical system we will study is given by Eq. (1):

$$\frac{d\theta_j}{dt}=E+\sin(\alpha_j-\theta_j)+\frac{K}{N}\sum_{i=1}^{N}\sin(\theta_i-\theta_j),\qquad j=1,2,\ldots,N\ . \tag{1}$$

The system (1) is formally very similar to the model studied by Fisher[5] and Sneddon,[4] with the exception of the final term: the coupling between phases in (1) is periodic in the phase difference, rather than linear. Also, the physical interpretation of the phases θ_j is somewhat different than in these elastic-coupling models, as discussed below.

The phases θ_j in Eq. (1) represent the phases of CDW domains; $E>0$ represents the applied dc field and $K>0$ is the strength of coupling between domains. The α_j represent the preferred phase of each domain, taken to be randomly distributed on $[0,2\pi]$. The strength of pinning is assumed to be constant for each domain, and has been normalized to one. The values E and K thus represent the strengths of the coupling and applied field relative to pinning. The time scale similarly reflects this normalization. For all-to-all coupling in the large-N limit, a random distribution of pinning phases is equivalent to an evenly spaced distribution $\alpha_j = 2\pi j/N$, $j = 1, 2, \ldots, N$.

In the phase-slip model (1), the θ_j represent the phases of entire CDW domains, or subdomains,[17] separated by ultrastrong pinning sites. In this sense, our phase variables have a similar interpretation to those in the model of Tua and Zawadowski.[24] Physically, the dynamics of Eq. (1) represent a competition between the energy in the applied field, the large pinning energy, and the energy of CDW amplitude collapse at the pinning sites. Phase distortions within a single domain due to weak pinning are not included in this model.

By describing a phase-slip process at the pinning sites by a phase-only model, we have neglected the dynamics of amplitude collapse, except as it is reflected by a periodic coupling term. Inui *et al.*[16] recently presented a detailed analysis showing how phase slip (with amplitude collapse) can be implicitly included in a phase-only model of switching CDW's in the limit of fast amplitude-mode dynamics. Zettl and Grüner[7] suggested that phase slip in switching CDW's could be accounted for by a sinusoidal coupling term. The model presented here is a discrete-site mean-field version of the phase-slip process, in the spirit of Fisher's treatment.[5] By choosing a particularly simple form for the periodic coupling function—but one with the right overall behavior—we are able to analyze much of the model's dynamics.

Previously,[22,23] we have shown that the large-N, mean-field version of (1) has a discontinuous and hysteretic depinning transition as the applied field E is varied. The switching seen in this model is in contrast to the smooth, reversible depinning which occurs in the corresponding equations with elastic phase coupling.[4,5] The threshold field $E_T(K)$ where the pinned solution, $\theta_j = \alpha_j + \sin^{-1}(E)$, becomes unstable to the formation of a coherent moving solution was shown[22,23] to be $E_T(K) = (1 - K^2/4)^{1/2}$ for $K < 2$ and $E_T(K) = 0$ for $K > 2$. At this threshold, the pinned solution bifurcates from a stable node to a saddle point in configuration space.[23]

B. Delayed switching

We now consider the time evolution of the system (1) during depinning. In order to simulate the experimental procedure where delayed switching is seen, we "apply" a superthreshold field $E > E_T(K)$ at $t = 0$ to the system (1) starting in the $E = 0$ pinned state $\theta_j = \alpha_j$. The response of the system depends on the value of E: for $E > 1$ the phases quickly leave the $E = 0$ pinned state and organize into a coherent moving solution. There is no delayed switching in this case. For $E_T(K) < E < 1$ the phases leave the pinned state quickly, but come to a near standstill at the saddle point $\theta_j = \alpha_j + \sin^{-1}(E)$, where they linger for a long time before finally leaving—again, very quickly—along the unstable manifold of the saddle point to form a coherent moving solution. Close to threshold, the time spent in the vicinity of the saddle point is much longer than any other part of the depinning process, resulting in a long delay before a rapid switch to the moving state. In this subsection we analyze the dynamics of (1) near the saddle point and derive an expression for the switching delay.

To characterize the collective state of the phases, we define the complex order parameter $re^{i\Psi} \equiv (1/N)\sum_j e^{i\theta_j}$. In the limit $N \to \infty$, the distribution of pinning phases α_j becomes continuous on $[0, 2\pi]$ and the phase θ_j can be written as a continuous function θ_α parametrized by the pinning phase α. In this limit the order parameter is given by

$$re^{i\Psi} = \frac{1}{2\pi} \int_0^{2\pi} e^{i\theta_\alpha} d\alpha \ . \tag{2}$$

We find numerically that for evenly spaced pinning sites the model is quite insensitive to the choice of N for all $N \geq 3$. In the infinite-N limit, an evenly spaced distribution of pinning sites becomes equivalent to a random distribution, but at finite N, simulations with a random distribution of pinning sites showed a much stronger finite-size effect than those with evenly spaced pinning sites. The insensitivity to N for evenly spaced pinning provides a useful trick allowing us to numerically investigate the large-N behavior of Eq. (1) using relatively small simulations (typically $N = 300$). Simulations with evenly spaced pinning sites did show a slight size dependence, especially at very small initial coherence ($r_0 < 10^{-8}$), and care was taken in using a sufficiently large system to eliminate any measurable dependence on N. The excellent agreement between the simulations and the analysis shows that the dynamics of Eq. (1) are well approximated by the infinite-N limit. Analysis of Eq. (1) will henceforth treat the case $N \to \infty$. Justification for applying the large-N limit to real CDW systems will be discussed in Sec. III.

Physically, the magnitude of the order parameter r $(0 \leq r < 1)$ characterizes phase coherence between CDW domains. In a pinned configuration, for example, where the phases of the domains are determined by a random locally preferred phase, there is no coherence among the domains; accordingly, all stable pinned solutions of (1) have $r = 0$. In the steady sliding state $(d\Psi/dt > 0)$ the model shows a large coherence between domains; all stable sliding solutions of (1) have $r \sim 1$. The rate of change of the order-parameter phase, $d\Psi/dt$, corresponds to the current carried by the CDW. The delayed switching observed experimentally corresponds to a delay in the current carried by the CDW. In our model, the onset of a "current" $(d\Psi/dt > 0)$ and the onset of coherence $(r \sim 1)$ occur simultaneously, as seen in Fig. 1.

The slowest step in the depinning process for $E_T(K) < E < 1$ is the evolution near the saddle point $\theta_\alpha = \alpha + \sin^{-1}(E)$. As we have shown elsewhere,[23] an interesting feature of the dynamics of (1) is that *any* initially incoherent $(r \sim 0)$ configuration will be funnelled towards this saddle, and from there coherence and steady rotation will evolve. Thus the delay before switching for any incoherent initial state is approximately given by

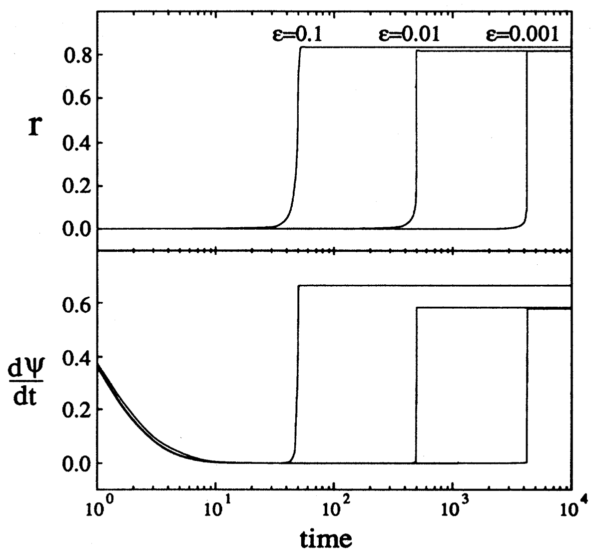

FIG. 1. Delayed onset of coherence r and current $d\Psi/dt$ for three normalized distances above threshold, $\varepsilon \equiv (E-E_T)/E_T = 0.1$, 0.01, and 0.0001. Note the logarithmic scale for the (dimensionless) time axis. Curves are from numerical integration of Eq. (1) with $N=300$, $r_0 \cong 5\times10^{-5}$, and $K=1$, giving $E_T=(\frac{3}{4})^{1/2}$. The initial state of the system was the $E=0$ pinned state with a small amount of random jitter: $\theta_j=\alpha_j+\eta_j$ with $\eta_j \sim O(10^{-2})$. The large values of $d\Psi/dt$ at $t<10$ show the rapid evolution from the initial state to the saddle point $\theta_j=\alpha_j+\sin^{-1}(E)$. Note that for a given ε the order parameter r and the rotation rate $d\Psi/dt$ switch after the same delay.

the time for coherence to develop starting near $\theta_\alpha = \alpha + \sin^{-1}(E)$.

Because the depinning process with $E>0$ changes an initially static configuration ($d\Psi/dt=0$) into a uniformly rotating configuration ($d\Psi/dt>0$), the analysis is greatly simplified by working in a coordinate system that is corotating with Ψ. In terms of the corotating phase variables defined as $\phi=(\theta-\Psi)$ and $\gamma=(\alpha-\Psi)$, the depinning transition appears as a Landau-type symmetry-breaking transition from the unstable equilibrium at $r=0$ to a stable, *static* equilibrium at $r\sim1$. Recasting the dynamical system (1) in terms of γ and ϕ_γ and writing the coupling term in (1) in terms of the order parameter gives the following mean-field equation:

$$\frac{d\Psi}{dt}\left[1-\frac{\partial\phi_\gamma}{\partial\gamma}\right]+\frac{dr}{dt}\frac{\partial\phi}{\partial r}=E+\sin(\gamma-\phi_\gamma)-Kr\sin(\phi_\gamma)\ . \tag{3}$$

In deriving Eq. (3) we have assumed that ϕ_γ only depends on r and γ. Assuming this dependence is equivalent to assuming that as the system leaves the saddle point $\phi_\gamma=\gamma+\sin^{-1}(E)$ it will not be free to visit all of state space, but is constrained to lie only in the unstable manifold of the saddle. Within the unstable manifold, two quantities are sufficient to characterize the state of the entire system: γ, which reflects the direction in which rotational symmetry has broken, and r, which reflects the position along the unstable manifold.

We now expand ϕ_γ about the saddle point $\phi_\gamma=\gamma+\sin^{-1}(E)$ in a Fourier series:

$$\phi_\gamma=\gamma+\sin^{-1}(E)+\sum_{k=1}^{\infty}A_k\sin(k\gamma)+\sum_{k=0}^{\infty}B_k\cos(k\gamma)\ , \tag{4}$$

where the Fourier coefficients A_k and B_k only depend on r. We assume that for small r each A_k and B_k can be expanded as a power series in r. We then solve Eq. (3) using a solution of the form of Eq. (4) with A_k, B_k, dr/dt, and $d\Psi/dt$ expanded in powers of r. This procedure gives a solution for ϕ_γ at each order of r. For self-consistency, these solutions must also satisfy the definition of the order parameter, which requires

$$\frac{1}{2\pi}\int_0^{2\pi}\cos(\phi_\gamma)d\gamma=r,\quad \frac{1}{2\pi}\int_0^{2\pi}\sin(\phi_\gamma)d\gamma=0\ . \tag{5}$$

Retaining terms to third order in r for $k\le2$ uniquely determines (after much algebra) the evolution equation for the coherence:

$$\frac{dr}{dt}=\left[\frac{K-K_T}{2}\right]r+\left[\frac{6-K_T^2}{2K_T}\right]r^3+O(r^5)\ , \tag{6}$$

where $K_T\equiv2(1-E^2)^{1/2}$. The form of the $O(r^3)$ coefficient in Eq. (6) was derived assuming that the system is close to threshold, that is, assuming $K-K_T \ll K_T$.

The value of r where the two terms on the right-hand side of Eq. (6) are equal, defined as $r^*\equiv[K_T(K-K_T)/(6-K_T^2)]^{1/2}$, marks a crossover point in the evolution of coherence. For $r(t)<r^*$, the cubic term is negligible and r grows as a slow exponential: $r(t)\cong r_0\exp(\sigma t)$, where $\sigma\equiv(K-K_T)/2$. Note that $\sigma\to0$ as $E\to E_T$. After $r(t)$ reaches the value r^*, the cubic term in Eq. (6) dominates the linear term and r grows very rapidly. The rapid onset of coherence is accompanied by the simultaneous rapid growth of $d\Psi/dt$, as seen in Fig. 1. We identify the switching delay $\tau_{\rm switch}$ as the time the system takes to evolve from r_0 to r^* by slow exponential growth: $r^*\equiv r_0\exp(\sigma\tau_{\rm switch})$. Solving for $\tau_{\rm switch}$ gives

$$\tau_{\rm switch}=\frac{1}{K-K_T}\ln\left[\frac{K_T(K-K_T)}{(6-K_T^2)r_0^2}\right]\ . \tag{7}$$

The switching delay given by Eq. (7) agrees very well with numerical integration data for all values of $E_T<E<1$ and $r_0<r^*$. Figure 2 compares numerical data with Eq. (7) as a function of the normalized distance above threshold $\varepsilon\equiv(E-E_T)/E_T$. Numerical data were obtained using fourth-order Runge-Kutta numerical integration of Eq. (1). The pinning phases were evenly spaced on $[0,2\pi]$ and the initial phase configuration was the $E=0$ pinned state plus a repeatable random jitter: $\theta_j=\alpha_j+\eta_j$ with $\eta_j\sim O(10^{-2})$, giving $r_0\cong1\times10^{-4}$ at each ε. The random jitter η_j is introduced to break the symmetry of the unstable equilibrium at $r=0$, which is present for infinite N and also for finite N with evenly spaced pinning. In the absence of any initial jitter, r_0 is

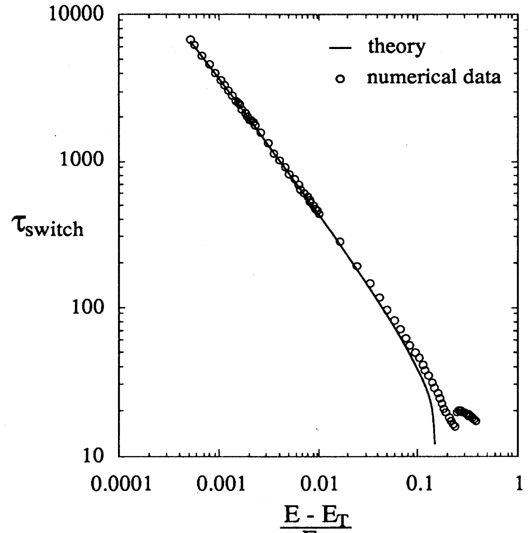

FIG. 2. Dependence of the delay (in dimensionless units) on the normalized distance above threshold $\varepsilon \equiv (E - E_T)/E_T$ for $K=1$. Curve shows the theory from Eq. (7); circles are from numerical integration of Eq. (1) with $N=300$. The initial state was the $E=0$ pinned state with random jitter: $\theta_j = \alpha_j + \eta_j$ with $\eta_j \sim O(10^{-2})$. The same initial state was used for all values of ε. The value of r_0 used to calculate the theoretical curve was taken directly from the numerical data as the smallest value of r during its evolution; the minimum r depended very slightly on ε, and a single value of $r_0 = 1\times10^{-4}$ was used in Eq. (7). The disagreement between theory and numerics at $\varepsilon > 0.1$ is due to the small but finite time taken for the *other* parts of the depinning process besides the time spent lingering near the saddle point.

zero and the predicted switching time is infinite from Eq. (7). An interesting detail is that during the very early evolution, as the system evolves from the initial configuration $\theta_j = \alpha_j + \eta_j$ towards the saddle point (before the delay), r actually decreases—that is, the system becomes *less* coherent as it approaches the saddle. Because of this effect, the appropriate value of r_0 to use in Eq. (7)— and the value used for the theory in Fig. 2— is not the initial r, but the *minimum r*, which is slightly less than the initial value. The switching event for the numerical data was defined as the time where $r(t)$ reached 0.75. Because of the rapidity of the switch, any other reasonable definition of the switch time would have given nearly identical results.

III. DISCUSSION OF THE MODEL AND ITS PREDICTIONS

The phase-slip model of delayed switching presented here is a highly simplified treatment of CDW dynamics. Several approximations made in the interest of keeping the model analytically tractable are known to be physically unrealistic, including the all-to-all coupling of domains and the uniform coupling and pinning strengths. Possible justifications for these assumptions have been discussed elsewhere.[5,22]

An unphysical aspect of the model is the absence of multiple pinned configurations. Metastable pinned states are seen experimentally in both switching and nonswitching systems,[25] as well as in models with elastic coupling.[5,26] The absence of multiple pinned states in our model is a direct result of the periodic coupling, which does not allow large phase differences between domains to build up. Our assumption is not a necessary feature in modeling phase slip: an alternative phase-slip model that does allow a large buildup of phase difference before phase slip begins has been proposed by Hall *et al.*[12,14,16]

A subclass of switching samples, termed "type II" by Hundley and Zettl,[17] shows multiple depinning transitions as the applied field is swept. In contrast, type-I switching samples[17] show a single, hysteretic switch. Because the parameters in our model are uniform, we do not see multiple switching; our model always behaves like a type-I sample. The effects of distributed parameters in our model remains an interesting open problem.

Because we interpret phases as entire domains, our use of a large number of phases might be questioned in light of recent experiments identifying a small number of coherent domains separated by phase-slip centers.[11,13,17] We believe the large-N treatment is justified: Several experiments on switching samples indicate that even when a small number of coherent domains can be identified, these large domains have been formed by the collective depinning of many subdomains. The relevant dynamical process leading to switching and delayed conduction in this case is the *simultaneous depinning of many subdomains* within a single large domain. Experiments by Hundley and Zettl,[17] for example, show that a switching sample will depin smoothly when subdomains are forced to depin individually rather than collectively by applying a temperature gradient across the sample.

In spite of these limitations and the model's simplicity, we find that several aspects of delayed switching seen experimentally are produced by the mean-field phase-slip model. These similarities include the following.

(1) For $\varepsilon \equiv (E - E_T)/E_T \ll 1$ and $r_0 < r^*$, the switching delay in the model is approximately related to ε by the power law

$$\tau_{\text{switch}} \propto \varepsilon^{-\beta}, \quad \beta \sim 1 \ . \tag{8}$$

This behavior is clearly seen at small ε in Fig. 2. This dependence is different from that predicted by other models of delayed switching,[12,16–21] as will be discussed elsewhere.[27] Experimentally, power-law behavior with exponent $\beta \sim 1$ at small ε is consistent with the data of Maeda *et al.* (Fig. 5 of Ref. 9).

(2) Above a certain value of ε, defined as ε_0, the depinning transition in our model is not delayed. The value of ε_0 is defined by the condition $E=1$. Similar behavior was seen experimentally by Zettl and Grüner,[7] who report that, for a bias current exceeding 1.25 times the threshold current, switching occurred without measurable delay. Measuring ε_0 in a switching sample will uniquely determine the appropriate value of K to be used in our model according to the formula $K = 2[1-(\varepsilon_0+1)^{-2}]^{1/2}$.

(3) For values of ε slightly below ε_0, the switching delay in the model decreases more rapidly than the power

law (8), giving a concave-downward shape to a log-log plot of $\tau_{\rm switch}$ versus ε, as seen in Fig. 2. This feature is also seen in the experimental data of Maeda *et al.* (Fig 5, Ref. 9).

(4) The switching delay in our model depends on an initial coherence. In Eq. (7) this dependence appears as the r_0 term in the logarithm, indicating that a sizable initial coherence will shorten the delay. For a sufficiently large initial coherence $r_0 > r^*$, Eq. (7) is not applicable because the cubic term in Eq. (6) will dominate throughout the evolution, leading to an extremely short switching delay. A reduction of the switching delay due to an organized initial state has been observed indirectly in the experiments of Kriza *et al.*[8] Using two closely spaced superthreshold pulses, Kriza *et al.*[8] were able to reduce the switching delay for the depinning which occurred during the second pulse; the shorter the spacing between the pulses, the shorter the observed switching delay.

(5) The delay in our model is a deterministic function of E, K, and r_0; we do not predict a scatter in the observed switching delay near threshold. Probabilistic models of depinning, for example, the model of Joos and Murray,[21] predict a scatter of delay times. A sizable scatter was reported by Zettl and Grüner;[7] more recently, Kriza *et al.*[8] attributed all scatter in the measured delay to "instrumental instability." It is not clear whether the scatter seen experimentally is an artifact or an intrinsic feature of delayed switching.

In conclusion, we have analyzed a very simple model of CDW domain dynamics with phase slip, and have found several features seen experimentally in switching samples. The results suggest that switching, hysteresis, and delayed onset of conduction are closely related phenomena which appear together when phase slip between domains occurs. Further experiments on delayed switching would be very useful to test in greater detail the predictive power of such a simple model.

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