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Mathematicians Prove That It's a Small World

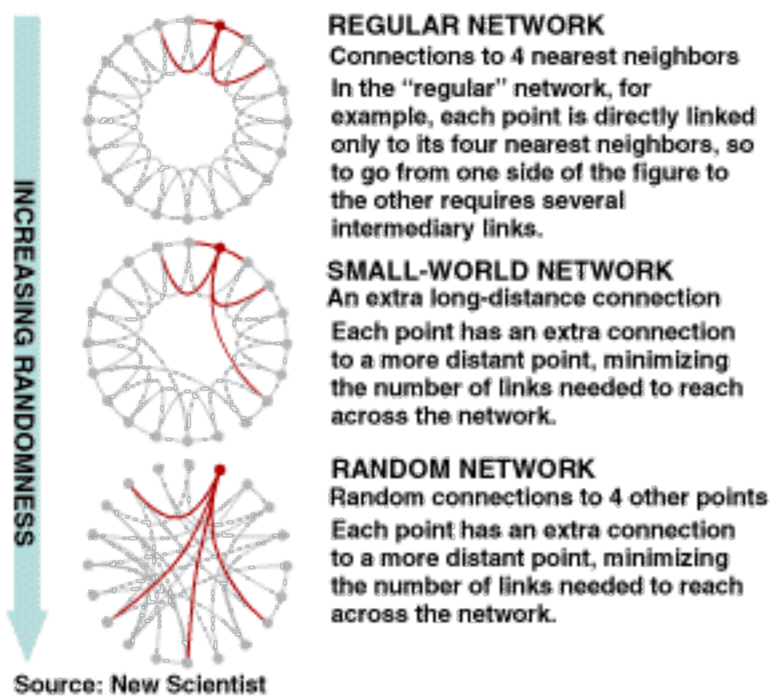
By SANDRA BLAKESLEE

NEW YORK -- Mathematicians have found a simple explanation for how big worlds can be turned into small worlds, a phenomenon summed up in the popular belief that anyone on earth can be linked to anyone else by a chain of only six other people -- the so-called six degrees of separation.

Any large set of linked, dynamic components -- be they people, electric power stations, brain cells or whatever -- can be transformed into a small world, the mathematicians said, by introducing short cuts between a few components. Relatively few short cuts can make big changes in a network, linking clusters of people, power stations or brain cells together in unexpected ways.

Separation, by Various Degrees

The smallness of a world or network can be expressed mathematically by the number of steps it takes to get from one element of it to any other. This depends on the degree of regularity of a network's interconnections.



Among people, the short cuts are well-connected individuals -- those who know many people from other countries, different disciplines or unusual subcultures. Such highly connected people establish surprising interconnections between groups that seem impossibly distant from one another, like Iowa farmers and Kalahari bushmen. For example, sometimes a farmer's son will join the Peace Corps in Africa and thus unexpected links are born.

This kind of interconnectedness is familiar to social animals like humans but now, for the first time, mathematicians have quantified it in a general model.

The finding, which is the first mathematical explanation of the small world phenomenon in dynamical systems, was reported in the June 4 issue of the journal Nature. The mathematical experiments were done by Steven Strogatz, a mathematician at Cornell University, and Duncan Watts, a postdoctoral fellow in the social sciences at Columbia University.

"They have made a great contribution to understanding small world effects," said James Collins, a mathematician at the Center for Biodynamics at Boston University. "The phenomenon has been mostly studied by sociologists and psychologists, but the mathematics community had not explored these networks in terms of self-organizing systems."

Such systems include social networks, technologies like the Internet and electric power grids, and life in general -- the fact that a sperm can penetrate an egg and set into play a series of biological processes that give rise to complex creatures from mere specks of protoplasm.

The finding that short cuts make all the difference explains why total strangers can quickly determine that they have acquaintances in common, why epidemics spread so rapidly in the modern world and may be useful in improving communication systems like cellular telephones and the Internet.

"There's a unifying mechanism in nature that makes things small and interconnected," Strogatz said.

But could it be modeled mathematically? To find out, the researchers focused attention on two extreme types of networks. One is a simple geometric lattice in the shape of a ring, with each node connected locally to its neighbors. Like a pure crystal, this lattice is highly ordered. The distance from one node to the next is characteristically long, and nodes tend to cluster tightly in neighborhoods.

The second model is a ringed lattice with the nodes connected randomly. The distance from one node to the next is characteristically shorter, but because everything is connected helter-skelter, there is very little order and nodes do not fall into common clusters.

Most biological, technological and social networks lie between these two extremes of ordered versus random connections. "We asked, what would happen if you take an ordered lattice and rewire some of the nodes, introducing short cuts?" Watts said. "You'd be making a regular network slightly more random by connecting nodes that would not normally be connected."

For example, in a regular network each node is connected only to its immediate neighbors -- like people in a rural village who rarely leave home and only know the people next door. But if one of those villagers travels to the big city, adding one distant link to all the close links at home, the village is connected to the wider world.

The results were astonishing, Watts said. The properties of the lattice change dramatically. It still forms clusters, but the pathways between clusters are much shorter. Immediate neighborhoods and more distant neighborhoods are suddenly closer together. Information spreads more quickly. Each node has access to more information and can do more things. In being connected to more neighbors, nodes begin to cooperate and synchronize.

The huge surprise is that it takes very few short cuts between nodes and clusters to turn a big world into a small world, Watts said. After a point, adding more short cuts does not improve network efficiency.

To find out if their abstract mathematical model applied to the real world, the researchers looked at three systems whose elements have been mapped completely -- the power transmission grid of the western United States, the 282 neurons that make up the nervous system of a simple worm called Caenorhabditis elegans, and the nearly 250,000 actors listed in the Internet Movie Database.

Each is a small world whose elements are closely connected to neighboring elements -- one power station to the next, one cell to another and one actor to another actor who tends to be cast in the same kind of films, Watts said.

Such worlds could be random but they are not. They have a great deal of structure, which is created by short cuts between a few power stations, a few special cells in the nervous system or certain prominent actors who make a lot of movies, Watts said. These short cuts can be identified and measured, which means that they act as a unifying principle for small worlds in general.

The finding of what difference a few short cuts can make has radical implications for people, Watts said. Most people know only their immediate neighbors and friends but a few people are well-connected. They cross geographic, social and political boundaries and make the world seem smaller.

For example, if you knew Secretary of State Madeline Albright, you would be connected to everyone she knows, including kings and queens. But even if you are not her friend, you may be connected to her indirectly. Perhaps your neighbor's daughter-in-law was Ms. Albright's college roommate. If so, you would be connected to the Queen of England by three degrees of separation.

The notion that all people are connected through indirect networks was first studied by the Stanley Milgram, a social psychologist at Harvard University who in 1967 asked people in Nebraska and Kansas to mail a letter to two people in Boston with whom they were not acquainted. They were to forward the letter to friends who might know the Boston people.

The average number of intermediaries between the Midwesterners and the Bostonians was six, a finding that was popularized in the 1990 in the play "Six Degrees of Separation," by John Guare. The play and a subsequent movie were based on the notion that everyone in the world is connected to everyone else through a chain of at most six acquaintances. If two people have one mutual acquaintance they have one degree of separation. Subsequent degrees of separation are based on the number of people needed to make the link.

A few years ago, college students embraced a game called "Six Degrees of Kevin Bacon," in which they linked the actor to other actors through as few links as possible. The Oracle of Bacon at Virginia, a Web site created by Brett Tjaden, now links 322,095 actors to Bacon, assigning each a Bacon score. Because Bacon has appeared in so many films, most actors have low Bacon scores.

The game can also be played with a relative newcomer like Leonardo DiCaprio. For example, DiCaprio was in "Les Cent et Une Nuits" (1995) with Harrison Ford, who was in "Apocalypse Now" (1979) with Marlon Brando, who was in "Countess from Hong Kong" (1967) with Charlie Chaplin. Thus, Chaplin has a DiCaprio score of three.

Short cuts can have positive or negative effects on society, Watts said. When human populations were isolated centuries ago, disease epidemics would be localized. But add jet travel -- a short cut -- and the world becomes smaller. Many experts believe that AIDS was first spread to the United States from Africa through an infected airline attendant who introduced the virus into gay bathhouses in New York. "Infectious diseases spread more quickly and easily in a small world, and it is alarming how few short cuts are needed to make the world small," Strogatz said.

On the other hand, short cuts can make networks more efficient. One might improve the efficiency of cell phone networks by deliberately introducing a few random connections between nodes, or phones, Collins said. That alone might improve traffic flow without building new relay stations. Similarly, one might improve the flow of information through the Internet by introducing a few random links between backbone computers on the Internet, he said. That might reduce the time needed to send e-mail or to find a Web site.

Other Places of Interest on The Web

- The Oracle of Bacon at Virginia.
- The Erdos Number Project.
- Six Degrees
