Keeping the Beat

Perfect synchrony in arrays of clocks and other oscillators

By IVARS PETERSON

Late in the winter of 1665, an ailing Christiaan Huygens was confined to his room for a few days. The Dutch physicist whiled away the hours of his confinement by closely observing and pondering the remarkable behavior of two pendulum clocks he had recently constructed.

Huygens noticed that the pendulums of the two suspended clocks, hanging side by side from a common support, were swinging in perfect synchrony. When one pendulum swung to the left, the other went to the right. The pendulums remained precisely in opposite phase for as long as he cared to watch.

His curiosity piqued, Huygens began to experiment. He deliberately disturbed the motion of one pendulum so that it no longer mirrored the other's movements. Within half an hour, the two pendulums were back in opposite-phase synchrony.

Huygens suspected that the clocks were somehow influencing each other, perhaps through air currents or vibrations of their common support. To test this notion, he moved the clocks to opposite sides of the room. The clocks gradually fell out of step, with one clock losing 5 seconds a day compared to the other. The two pendulums no longer swung at exactly the same frequency or in opposite phase.

Conversely, as long as the two clocks interacted in some way not yet fully determined, they kept precisely the same time, even though they were not identical. In general terms, what Huygens had serendipitously discovered was a phenomenon that came to be known as the mutual synchronization of coupled oscillators.

Mutual synchronization can occur in rows of pendulums (SN: 12/9/95, p. 389), certain types of electrical circuits, and arrays of superconducting devices known as Josephson junctions. Similar patterns of behavior also appear in the synchronized flashes displayed by some species of fireflies (SN: 8/31/91, p. 136), the chorusing of crickets, the coordinated beating of rhythm-setting cells in the heart, and voltage fluctuations in networks of neurons in the brain (SN: 1/18/92, p. 39).

In each case, nonidentical oscillators interact to beat with a single rhythm. "This phenomenon lets a community be a more precise oscillator than you could ever expect to get from the individual elements," says Kurt Wiesenfeld of the Georgia Institute of Technology in Atlanta.

In such groups, fast oscillators get slowed down and slow ones get speeded up to lock the entire ensemble into a compromise rate.

"A lot of people are really interested in understanding the physics of phase locking, and there's a lot of physics to understand," says Samuel P. Benz of the National Institute of Standards and Technology in Boulder, Colo.

Benz and his coworkers, for example, take advantage of phase locking to ensure that large arrays of superconducting Josephson junctions display voltage oscillations of a well-defined frequency, despite small differences among the individual components. Such arrays are useful as components of detectors of electromagnetic radiation in the submillimeter range—wavelengths of interest in radioastronomy and atmospheric pollution monitoring.

Over the years, researchers have formulated equations to capture the essential features of individual oscillators, both physical and biological. With such mathematical models, they have tried to predict oscillator behavior in a variety of settings, including the conditions that lead to synchronization.

Creating the right equations to describe systems of oscillators and then solving the equations to make testable predictions has proved a difficult, if not impossible, task. Recently, researchers neatly connected theory and experiment for one interesting case.

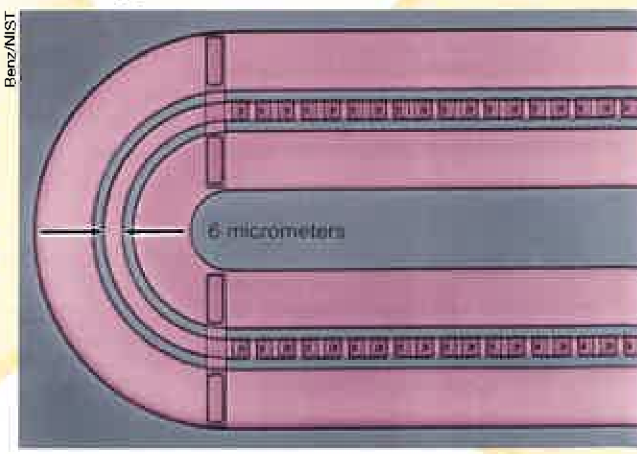
Physicists Wiesenfeld and Pere Colet of Georgia Tech, along with mathematician Steven H. Strogatz of Cornell University, uncovered a direct, previously unsuspected link between the mathematics describing a string of Josephson junctions and a 1975 set of equations called the Kuramoto model. The simple, solvable equations of this oscillator model had elucidated some aspects of the behavior of abstract coupled systems, but originally it appeared to have no connection with any realistic physical or biological system of oscillators.

"It was a solution waiting for a physical realization," Strogatz remarks.

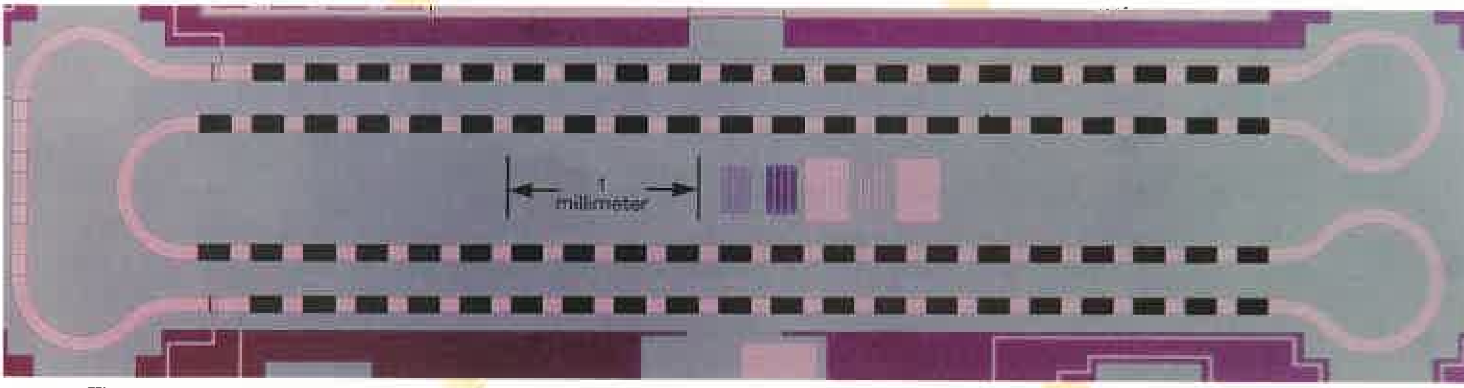
By establishing the link, researchers can now quantitatively predict under what circumstances a row of Josephson junctions can synchronize voltage fluctuations and how well matched the individual devices must be in terms of their physical characteristics in order to become fully synchronized.

Wiesenfeld and his colleagues reported these findings in the Jan. 15 PHYSICAL REVIEW LETTERS.

Physicist Yoshiki Kuramoto of the University of Kyoto in Japan was inspired to create his mathematical oscillator model by the work of Arthur T. Winfree, now at the University



A bend interrupts a series array of Josephson junctions suitable for equipment to maintain voltage standards.



P.A.A. Booi and Benz/NIST

This array of Josephson junction oscillators generates 0.16 milliwatt signals at a frequency of 240 gigahertz. Each black rectangle represents a group of 24 Josephson junctions.

of Arizona in Tucson. In 1967, Winfree suggested that the onset of synchronization in large populations of oscillators is analogous to a phase transition, such as the freezing of a liquid.

Initially, Winfree proposed, the oscillators behave incoherently, with each one running at its natural frequency. When the spread of frequencies among the oscillators falls below a certain critical value, the system spontaneously "freezes" into synchrony.

Kuramoto came across Winfree's paper in 1975, just as he was shifting his research from the statistical physics of phase transitions to rhythmic phenomena such as oscillating chemical reactions (SN: 7/1/89, p. 6) and circadian clocks. "It had a great impact on me," Kuramoto says.

Having already worked out a coupled oscillator model to account for some of the swirling, cyclic patterns observed in a chemical system known as the Belousov-Zhabotinsky reaction, Kuramoto extended this mathematical framework to incorporate Winfree's ideas.

The result was a set of differential equations with two components. One term describes what the oscillators do individually. Such an abstract oscillator can be imagined as a point moving at a certain rate along the circumference of a circle. This oscillator has a phase (where the point is in its cycle) and a period (how long the point takes to go around).

A second term describes how these oscillators influence each other to slow down speeders and speed up slowpokes. To make it possible to solve the full set of equations, Kuramoto specified that every oscillator interacts equally with every other oscillator in the array, not just its nearest neighbors.

Although the Kuramoto model did not correspond to any particular physical or biological system, it nonetheless became an attractive test case on which theorists could try out various hypotheses about oscillator behavior.

"Whether [Kuramoto's model] is applicable in any particular setting or not, you really need to understand the different kinds of behavior [displayed in this simple model] before you can understand more complicated things," Wiesenfeld says.

A typical Josephson junction consists of two superconducting layers separated by a thin film of insulating material. Because pairs of electrons can tunnel quantum mechanically through the insulating layer, a sporadic current can flow from one superconductor to the other, even when there is no voltage difference between them.

By adding a battery to force a sufficiently strong current through a Josephson junction, one can cause the voltage generated in the device to oscillate. Under these conditions, a junction's voltage oscillates at a frequency that depends on such factors as the device's area and composition. In an array, these oscillators are connected and end up oscillating together.

The great stability of this collective oscillation frequency makes such arrays valuable for maintaining the U.S. standard for the volt. Josephson junctions can also be used to measure very small electric currents and magnetic fields.

Both Wiesenfeld and Strogatz were familiar with oscillator models in general and the theory underlying the behavior of Josephson junction oscillators in particular. With help from mathematician Jim W. Swift of Northern Arizona University in Flagstaff, they began by figuring out a way to simplify the Josephson equations in a special case.

"It was then that we noticed the link," Wiesenfeld says. The junction oscillator equations in that case were equivalent to those of the Kuramoto model.

"Though people had studied Josephson junctions since the 1970s, no one had recognized the Kuramoto model in this context before," he notes.

This discovery meant that the researchers could use the known solutions of the Kuramoto equations to predict how a string of similar, but nonidentical, Josephson junction oscillators becomes synchronized.

A system functioning according to those equations would exhibit two transitions as the current that is forced through it changes. At first, the oscillators act independently. Then the system shifts to an intermediate stage in which the oscillators are partially synchronized.

"Oscillators near the middle of the frequency distribution would be the first ones to lock together," Strogatz says. "A small group of oscillators happens to get synchronized, and it exerts a coherent influence on the rest of the population because it stands out above the background noise. So it recruits more oscillators, and you get a positive feedback process."

The second transition occurs at the final stage, when the last stragglers are locked into the group. "We can predict where the two transitions occur and what currents produce them," Strogatz emphasizes.

Kuramoto comments, "Until [I received] a preprint of the Wiesenfeld paper, I had not imagined that such a simple oscillator model could find any physical counterpart at such a quantitative level."

The calculations also show that experimenters should be able to test these predictions with existing technology, though no one has done so yet. In general, "Josephson junction arrays are excellent experimental systems for studying phase locking and related phenomena," Benz notes.

Now, Wiesenfeld and others are investigating whether the same conclusions apply to mathematical models that more completely describe Josephson junctions, taking into account subtle electrical effects in these circuits.

There is also great interest in the question of how synchronization occurs in two-dimensional grids rather than just rows of Josephson junction oscillators. "That's a big open question," Wiesenfeld says.

In this case, the assumption that every oscillator interacts with every other oscillator no longer applies. More complicated patterns, such as spirals and vortices of activity, come into play. Nonetheless, "I think that qualitatively similar [transition] phenomena will be there," Wiesenfeld asserts.

Finally, "it still remains to be seen whether any biological system is well described by the Kuramoto model," Strogatz says. "But it's nice that an application for it was found in a physical system." □