MATHEMATICS

All together now . . .

Ian Stewart

ONE of the most spectacular displays in the whole of nature occurs in South-East Asia, where huge swarms of fireflies flash in synchrony. As H. M. Smith (*Science* **82**, 151; 1935) described the phenomenon:

Imagine a tree thirty-five to forty feet high, apparently with a firefly on every leaf, and all the fireflies flashing in perfect unison at the rate of about three times in two seconds, the tree being in complete darkness between flashes. Imagine a tenth of a mile of river front with an unbroken line of mangrove trees with fireflies on every leaf flashing in synchronism, the insects on the trees at the ends of the line acting in perfect unison with those between. Then, if one's imagination is sufficiently vivid, he may form some conception of this amazing spectacle.

Why do the flashes synchronize? Renato Mirollo and Steven Strogatz (*SIAM Journal of Applied Mathematics* **50**, 1645–1662; 1990) have (the cliché is irresistible) shed a great deal of light on this problem by proving a mathematical conjecture due to C. S. Peskin. They show that synchrony is the rule for mathematical models in which every firefly interacts with every other.

Ivan Polunin/NHPA

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Firefly flashing — an "amazing spectacle".

It is natural to model the insects as a population of oscillators, coupled together by visual signals. In most previous work on the synchronization problem, the coupling between these oscillators is assumed to be smooth. But, in 1975, Peskin formulated a mathematical model in which the oscillators are coupled discontinuously (*Mathematical Aspects of Heart Physiology*; Courant Institute of Mathematical Sciences, New York). He conjectured that in this model, for almost all initial conditions, eventually all oscillators become synchronized. It is this conjecture that Mirollo and Strogatz have proved.

Peskin's mathematical model involves a large number of identical oscillators, with fully symmetric coupling — that is, such that each oscillator affects all of the others in exactly the same manner. The component oscillators are of the 'integrate-and-fire' type, in which a voltage-like variable increases up to a threshold value, and as soon as it reaches this threshold the oscillator 'fires' and the variable drops back to zero.

The most unusual feature of Peskin's model is that the oscillators are pulse-coupled. That is, an oscillator affects its neighbours only at the instant when it fires; and it then increases the appropriate variable in each neighbouring oscillator by a fixed amount, or causes it to fire if the variable would exceed the threshold. The mathematical difficulty is to disentangle all of these interactions. Mirollo and Strogatz analyse a class of models similar to, but more general than, the one proposed by Peskin. They prove that, no matter what the initial conditions are, eventually all of the oscillators become synchronized. The proof is based on the idea of absorption, which happens when two oscillators with different phases 'lock together' and thereafter stay in phase with each other. Because the coupling is fully symmetric, once a group of oscillators has locked together, it cannot 'unlock'. The idea is that a sequence of absorptions eventually locks all of the oscillators together, and a geometric and analytic argument is used to show that this "almost always" happens.

The authors remark that many other systems of biological oscillators tend to function in synchrony: "the pacemaker cells of the heart, networks of neurons in the circadian pacemaker and hippocampus, the insulin-secreting cells of the pancreas, crickets that chirp in unison, and groups of women whose menstrual periods become mutually synchronized". It now seems that all of these systems attain synchrony by the same mathematical mechanism: absorption in networks of pulse-coupled oscillators.

They also formulate a conjecture of their own: that synchrony is also the rule in any connected network of pulse-coupled oscillators, and not just in networks with fully symmetric coupling. In other words, the fireflies will synchronize even when some of them can't see some of the others, provided no isolated group exists that cannot see any of the rest. But in these more general networks absorptions can be 'unlocked' by subsequent interaction, so a new method of proof will be needed. Mathematicians remain in the dark about these more subtle aspects of firefly synchronization. □

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BUCKMINSTERFULLERENE

Sooty superconductors

Arthur W. Sleight

BUCKMINSTERFULLERENE, the soccerball-shaped allotrope of carbon, continues to yield surprises at a breathless pace. Barely weeks after it was revealed that doping films of C_{60} with potassium renders them conducting, we now learn on page 600 of this issue[1] that cooling the doped material to 18 K makes it superconducting. This remarkable, unanticipated finding can be regarded as forging a link between organic and inorganic superconductors.

Although the molecular structures of the principal fullerenes, C_{60} and C_{70}, are now well established, the structures of the solids obtained on condensing these molecules are only poorly known. The C_{60} films which were doped with potassium to become conducting and superconducting were amorphous[2].

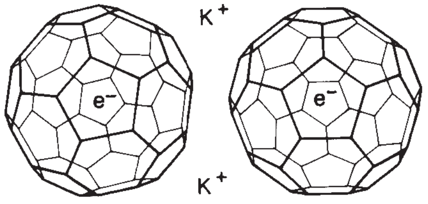

FIG. 1 Interstitial potassium atoms could give up an electron to the C_{60} molecules.

Nevertheless, well crystallized C_{60} and C_{70} have been prepared. In crystals, the pseudospherical molecules tend to take on either a cubic close-packed or hexagonal close-packed arrangement[3]. In the case of pure C_{60}, the cubic arrangement tends to be preferred. At room temperature, these pseudospheres rotate about their centres. Thus, although there is a periodic array of pseudospheres, there is not a well defined periodic array of carbon atoms. On cooling to 249 K, there is a first-order phase transition where at least much of this rotation apparently ceases[4].

Like graphite, C_{60} is expected to be a reasonable electron acceptor, and a charge transfer salt containing the C_{60}^- anion has been prepared[5] by electrolysis: $(Ph_4As^+C_{60}^-)(Ph_4As^+Cl^-)$. A team at AT&T Bell Laboratories also recently found[2] that when C_{60} is exposed to alkali-metal vapours, it becomes electrically conducting. Now this group has found superconductivity in several samples of K_xC_{60}. The actual composition of the superconductor is unknown but may be close to K_3C_{60}. The detailed structure of this phase is also unknown, but there is adequate space for potassium cations to be placed in-