The Fiscal Space Conjecture: Theoretical Reflections

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The nineteenth century carried to extravagant lengths the criterion of what one can call for short 'the financial results,' as a test of the advisability of any course of action sponsored by private or by collective action. The whole conduct of life was made into a sort of parody of an accountant's nightmare. Instead of using their vastly increased material and technical resources to build a wonder city, the men of the nineteenth century built slums; and they thought it right and advisable to build slums because slums, on the test of private enterprise, 'paid,' whereas the wonder city would, they thought, have been an act of foolish extravagance, which would, in the imbecile idiom of the financial fashion, have 'mortgaged the future' — though how the construction to-day of great and glorious works can impoverish the future, no man can see until his mind is beset by false analogies from an irrelevant accountancy... The same rule of self-destructive financial calculation governs every walk of life... We are capable of shutting off the sun and the stars because they do not pay a dividend... But once we allow ourselves to be disobedient to the test of an accountant's profit, we have begun to change our civilization.

John Maynard Keynes (June 1933)

INTRODUCTION

Recent applied work in fiscal policy (for example, Roy et al, Chapter 2 in this volume) has advanced the so-called 'fiscal space conjecture'.¹ This conjecture states that:
For any public investment programme, the more the public good characteristics of the public investment outputs, the less the precision and predictability of the fiduciary payback calculation. The less the public good characteristics, the more the precision and predictability of the fiduciary payback calculation.

And:

For any public investment programme, the more the public good characteristics of the public investment outputs, the more the precision and predictability of the development payback calculation. The less the public good characteristics, the less the precision and predictability of the development payback calculation.

Although the conjecture is suggestive, its terms have not been fully defined and the conditions under which it is justified have not been fully identified. Since the conjecture may provide a useful framework for thinking about an appropriate approach to public investment, it may be deemed important to determine under what conditions it would hold true. In an attempt to explore the problem, it is necessary to begin with some definitions.

**INITIAL DEFINITIONS: SOCIAL ACHIEVEMENTS, DEVELOPMENT PAYBACK AND FINANCIAL PAYBACK**

Suppose that a society consists of n persons, S = {1, ..., n}. Let us also suppose that there is an 'individual achievement space', A, composed of l distinct dimensions (1, ..., l) in relation to which cardinaly measurable achievements are defined, so that \( A \subseteq R^l_+ \). Each member of the society, \( i \in S \), possesses individual achievements \( a = (a_1, ..., a_l) \in A \), which together compose a social achievement vector \( a = (a_1, ..., a_n) \in A^n \). If a social achievement vector achieved in a given social state of affairs is designated by \( \Phi \), then by employing a corresponding superscript, it is possible to define a 'development ordering' by a reflexive and transitive binary relation, \( > \), over the set of possible social achievement vectors \( \{a^\Phi\} \). This development ordering reflects judgements about the manner in which social achievements are deemed to give rise to development. For simplicity, it will be assumed that \( > \) is complete and strictly monotonic in social achievements, so that it can be represented by a real-valued 'development evaluation function' \( D(a^\Phi) \) with the feature that higher values of the function correspond to greater development as defined by the relation \( > \). The development evaluation function simultaneously encapsulates judgements concerning the relative significance of distinct dimensions of individual achievements and the distribution of these achievements across persons.

It is assumed that one of the achievements (without loss of generality, achievement 1) is defined by 'income'. Each individual then possesses income, \( y_i = a_i \), and the total
income of the society is given by \( Y = \sum_{i=1}^{\infty} y_i \). As previously, the superscript \( \Phi \) may be employed to refer to the incomes prevailing in a given social state of affairs.

Now, define a public investment as an action that may be undertaken by a government (at some cost, \( c \)) which has the effect of causing a change from one social achievement vector, \( a^{\Phi_1} \), to another \( a^{\Phi_2} \). The effect of this public investment is to bring about a change, \( \Delta D = D(a^{\Phi_2}) - D(a^{\Phi_1}) \), in the level of development. This may be referred to as the development payback of the public investment. It is determined by the initial social achievement vector, \( a^{\Phi_1} \), the impact of the public investment on the social achievement vector (i.e. on \( \Delta a = (a^{\Phi_2} - a^{\Phi_1}) \)) and the treatment of this impact by the development evaluation function. As such, the development payback cannot be determined without a composite assessment which integrates empirical observations and normative judgements.

In contrast, the financial ("fiduciary" in the language adopted by Roy et al., Chapter 2) payback of the public investment may be defined as follows. The "societal financial payback" is \( \Delta Y = Y^{\Phi_2} - Y^{\Phi_1} \). If government imposes on each individual, \( i \in S \), a proportionate tax rate of \( \tau_i \) (0 \( \leq \tau_i \leq 1 \)), in a given social state of affairs, \( \Phi \) (designated as before by a superscript), then the "governmental financial payback" is \( \Delta Y = - \sum_{i=1}^{n} \tau_i D(a^{\Phi_1}) + \sum_{i=1}^{n} \sum_{j=2}^{n} \Phi_j D a_{ij} \).

It is evident that the governmental financial payback is less than or equal to the societal financial payback. Henceforth, the focus will be on the societal financial payback, although the arguments may be readily extended to the case of the governmental financial payback.

Note that the marginal development payback (for example, that produced by "small" public investment projects) can be written as follows:

\[
dD = \sum_{i=1}^{n} \sum_{j=1}^{l} \frac{\partial D}{\partial a_{ij}} + \sum_{i=1}^{n} \sum_{j=2}^{l} \frac{\partial D}{\partial a_{ij}} a_{ij}
\]

\[
= \sum_{i=1}^{n} \frac{\partial D}{\partial y_i} a_{ij} + \sum_{i=1}^{n} \sum_{j=2}^{l} \frac{\partial D}{\partial a_{ij}} a_{ij}
\]

Even if a public investment project were to consist only of a financial payback (i.e. \( \sum_{i=1}^{n} \sum_{j=2}^{l} \frac{\partial D}{\partial a_{ij}} = 0 \)), then the level of the development payback would still be potentially affected by the distribution of the societal financial payback across persons, since the financial payback, \( f_{ij} \), received by each individual is valued to an extent that depends on the marginal social valuation \( \frac{\partial D}{\partial y_i} \). In the special case where such distributional considerations play no role (either because the evaluator's normative judgements are distribution blind or because of the availability of lump sum tax and transfer instruments or similar means of achieving any desired distributional objectives), then:

\[
dD = \sum_{i=1}^{n} dy_i + \sum_{i=1}^{n} \sum_{j=2}^{l} \frac{\partial D}{\partial a_{ij}} a_{ij}
\]
that is, the marginal total development payback is equal to the marginal total financial payback plus the marginal non-financial development payback:

\[ dD = dTFP + dNFDP \]

The discussion is specialized to this case for simplicity in the subsequent analysis, although it is not essential to do so.

**PUBLIC GOOD CHARACTERISTICS OF PUBLIC INVESTMENTS**

Public investments often possess 'public good characteristics'. How is this feature of public investments best understood within the current framework? Given a public investment project, for each individual and achievement it is possible to write:

\[ \Delta a_y = (a_y^{02} - a_y^{01}) = \theta_y \delta \]

where \( \delta \) is the largest increase in the achievement experienced by any of the individuals.

Let the matrix \( \theta \), consisting of the coefficients \( \theta_{y} \), be called the impact coefficient matrix. These coefficients are assumed non-negative for simplicity. Consider a feasible public investment, P1, and suppose it has impact coefficient matrix \( \theta \). Now consider the set, \( J \), of impact coefficient matrices \( \theta' \) that is identical to \( \theta \) with the exception that its coefficient vector, \( \theta_{it} (i \in (1, \ldots, n)) \), is less than the corresponding coefficient vector \( \theta_{it} \) for one or more \( i \in (1, \ldots, n) \) and unchanged for the remaining \( j \in (1, \ldots, n) \). Every element of this set must also correspond to a feasible public investment project if the original public investment, P1, features complete excludability, and if the benefits of the public investment project are perfectly divisible (which is assumed). This is because it must be possible for such a project to partially or wholly 'shut off' the benefits accruing to any individual without diminishing the benefits that accrue to others, even if these benefits are, in the first instance, jointly supplied. Alternatively, if there is not complete excludability, then at least some elements of the set \( J \) will be unfeasible. At the opposite extreme of complete non-excludability, all elements of the set \( J \) are unfeasible, implying that the only way to provide the good undiminished to one person is to provide it undiminished to all. The 'proportion' of elements in the set that are unfeasible, defined appropriately, provides a measure of the excludability of the benefits of a public investment. Let \( \alpha \in [0, 1] \) be such a monotonically decreasing index of the degree of excludability of a public investment, such that \( \alpha = 1 \) when the benefits of the public investment are not excludable at all (corresponding to a public investment with maximal public good characteristics) and \( \alpha = 0 \) when they are completely excludable (corresponding to a public investment with minimal public good characteristics). This is a definition that focuses on the benefits delivered by alternative public investments rather than on their physical or infrastructural features.
What about the assumption of non-rivalry in consumption? If the public investment, $P_i$, is efficiently produced, and if there is no rivalry in consumption, then 'shutting down' (or more generally diminishing) the benefits received by any individual cannot increase the benefits accruing to any individual who continues to receive them. Thus, there is no feasible public investment that has a distinct impact coefficient matrix such that its coefficients are identical to those of $\theta_j$, with the exception that its coefficients vector $\theta_{ik} (k \in \{1, \ldots, l\})$ is less than the original coefficient vector for one or more $i \in \{1, \ldots, n\}$, and its coefficient vector is greater than or equal to the original coefficient vector for all other $i \in \{1, \ldots, n\}$ (and strictly greater for at least one). In contrast, if there is some rivalry in consumption, then there exist such feasible public investments. The magnitude of the trade-off between the lowered and the raised impact coefficients, appropriately defined, provides a measure of the extent to which there is rivalry in experiencing the benefits of a public investment.

Let $\beta \in [0, 1]$ be a monotonically decreasing index of the degree of rivalry present in a public investment, such that $\beta = 1$ when the public investment is such that there is no rivalry at all (corresponding to the case of maximal public good characteristics) and $\beta = 0$ when there is the greatest rivalry (corresponding to the case of minimal public good characteristics). This is, once more, a definition that focuses on the benefits delivered by alternative public investments rather than on their physical or infrastructural features.

It is important to note that whether a public investment possesses a high degree of public good characteristics does not appear to depend, in principle, on whether or not the payback generated by the public investment is predominantly financial or non-financial. Public investments that have paybacks of either type could vary in their public good characteristics. Of course, there may be an empirical relationship between the type of payback generated and the extent of public good characteristics. Whether this is the case would have to be empirically investigated.

**DEFINING PREDICTABILITY AND PRECISION**

Having presented these preliminaries, it is now possible to address the 'fiscal space conjecture' as described by Roy et al (Chapter 2). First, predictability and precision need to be formally defined. Roy et al define precision as the 'degree of expected error in ex ante calculations of payback' and predictability as the 'degree of observed error in ex post payback outcomes'. If the social decision-maker has 'rational expectations' the expectation of the difference between predicted payback and actual payback is zero. It seems unappealing and implausible to assume that this difference is either systematically negative or positive. It seems more plausible that Roy et al have in mind a measure that penalizes greater deviation between expected payback and actual payback (such as the variance of the errors in prediction). It will therefore be assumed that predictability and
precision both refer to such a measure, specifically the variance in the errors of prediction. Since the *ex ante* and *ex post* perspectives are equivalent in regard to such a measure, henceforth both terms can be referred to under the heading of 'predictability'.

**Predictability and Public Good Characteristics**

Having defined these terms, can it be concluded that they vary in any systematic way with the public good characteristics of public investments?

If \( dD = dSFP + dNFDP \)

then the errors in prediction of the development payback \( \varepsilon_D \) are defined by:

\[
\varepsilon_D = (E(dD) - dD) = E(dSFP) + E(dNFDP) - dSFP - dNFDP
\]

Further, predictability or precision of the development payback refers to:

\[
\text{var} \varepsilon_D = \text{var}(E(dSFP) + E(dNFDP) - dSFP - dNFDP)
\]

\[
= \text{var}(dSFP) + \text{var}(dNFDP) + 2\text{cov}(dSFP,dNFDP)
\]

whereas errors in prediction of the financial payback are defined by:

\[
\varepsilon_F = E(dSFP) - dSFP
\]

and predictability or precision of the financial payback refers to:

\[
\text{var} \varepsilon_F = \text{var}(dSFP)
\]

It is interesting to note that \( \text{var} \varepsilon_D - \text{var} \varepsilon_F = \text{var}(dNFDP) + 2\text{cov}(dSFP,dNFDP) \).

It follows that whether the predictability of the development payback is larger than that of the financial payback will depend jointly on \( \text{var}(dNFDP) \) and \( \text{cov}(dSFP,dNFDP) \). While it is straightforward to assume that the former is positive, the latter could, in principle, be of either sign, and would appear to depend on the exact nature of the interlinkages between financial and non-financial development achievements generated by the public investment.

In particular, given that \( \text{var}(dNFDP) \) is positive, the predictability of the development payback is greater than the predictability of the financial payback (\( \text{var} \varepsilon_D - \text{var} \varepsilon_F < 0 \)) if and only if the covariance between the non-financial development payback and
the financial payback is either positive or negative and sufficiently small (possessing a magnitude that is less than half that of the predictability of the non-financial development payback). Note that both terms refer to properties of the outcomes generated by the public investment as assessed by the development evaluation function. As such, they each depend jointly on the empirical outcomes of the public investment and the normative assessments of these outcomes implicit in the function.

Consider the first (sufficient) condition — that the covariance between the societal financial payback and the non-financial development payback of a public investment project is positive. Is this likely to be the case? Whether investment projects that generate positive (negative) societal financial payback generate positive (negative) non-financial development paybacks will depend on whether there exist causal interconnections through which higher private incomes are translated into better (non-financial) development outcomes or vice versa. Much of the literature on development strongly suggests that such causal linkages, or 'spill-over', not only exist, but are pervasive. It is clear that the ‘spill-over’ that is present in the development process is likely to be largest when the public good characteristics of the development process are highest. (When the public good characteristics of an investment project are high then, from the definitions given above, it is not possible to design the projects alternatively so as to restrict the benefits to selected persons, and the benefits to particular persons cannot be increased by decreasing those available to others. However, this is precisely the case in instances in which there exist spill-over from financial to non-financial development outcomes, or vice versa.) It is sufficient to think of specific empirical examples, such as the income generation benefits of improving public health and sanitation, to see that this is very frequently the case. In contrast, there may be projects in which the pursuit of higher developmental payback is associated with lower financial payback and vice versa. Consider, for instance, the degree of success in collecting health service user fees from poor users. These diverse examples suggest that the sign and magnitude of the covariance between different kinds of paybacks in a given type of investment project are empirical matters. In particular, the greater public good characteristics of a public investment project may or may not be associated with the possession of higher covariance between the development payback and the financial payback of the project, and such covariance need not necessarily be positive.

So far the relative magnitudes of two concepts (the predictability of the development payback and the predictability of the societal financial payback) associated with any single public investment project have been explored. This, although interesting, is not the subject of the ‘fiscal space conjecture’ that, instead, involves comparisons between investment projects of each of the two concepts (predictability of development payback and the predictability of societal financial payback) individually.
THE FISCAL SPACE CONJECTURE

It is now necessary to formally define the ‘fiscal space conjecture’. A first public investment project (P1) is defined as having ‘greater public good characteristics’ than a second public investment project (P2) if \((\alpha_{1}, \beta_{1}) > (\alpha_{2}, \beta_{2})\), where the subscripts denote the respective public investments and \((\alpha, \beta)\) are the measures of excludability and rivalry defined above. The fiscal space conjecture (FSC1) then states that if P1 has greater public good characteristics than P2 (i.e. \((\alpha_{1}, \beta_{1}) > (\alpha_{2}, \beta_{2})\)) then \(\text{var } \varepsilon_{D1} < \text{var } \varepsilon_{D2}\) and \(\text{var } \varepsilon_{F1} < \text{var } \varepsilon_{F2}\) where the subscripts 1 and 2 refer to the respective public investment projects.

It may be inappropriate to compare ‘unlike’ public investment projects, for instance those that have different levels of expected (overall) development payback. Taking this approach, it is possible to refine the fiscal space conjecture, for instance, as follows (FSC2): if P1 has greater public good characteristics than P2 (i.e. \((\alpha_{1}, \beta_{1}) > (\alpha_{2}, \beta_{2})\)) and P1 and P2 have the same expected overall development payback (i.e. \(E(\varepsilon_{D1}) = E(\varepsilon_{D2})\)), then \(\text{var } \varepsilon_{D1} < \text{var } \varepsilon_{D2}\) and \(\text{var } \varepsilon_{F1} > \text{var } \varepsilon_{F2}\) where the subscripts 1 and 2 refer to the respective public investment projects.

It may be argued that this is not a sufficiently stringent conception of likeness. However, the variance of the financial payback and that of the overall developmental payback cannot be meaningfully compared unless the expected levels of these paybacks are both similar. Accordingly, it is possible to present the stricter definition (FSC3): if P1 has greater public good characteristics than P2 (i.e. \((\alpha_{1}, \beta_{1}) > (\alpha_{2}, \beta_{2})\)) and P1 and P2 have the same expected overall development payback (i.e. \(E(\varepsilon_{D1}) = E(\varepsilon_{D2})\)), and the same expected financial payback (i.e. \(E(\varepsilon_{DFP1}) = \sum_{i=1}^{n} E(\varepsilon_{Dy1}) = \sum_{i=1}^{n} E(\varepsilon_{Dy2}) = E(\varepsilon_{DFP2})\)), then \(\text{var } \varepsilon_{D1} < \text{var } \varepsilon_{D2}\) and \(\text{var } \varepsilon_{F1} > \text{var } \varepsilon_{F2}\) where the subscripts 1 and 2 refer to the respective public investment projects.

It may be noted that, if the expected overall development payback and the expected financial payback are the same, then it follows that the expected non-financial development payback is also the same. That is:

\[ E(\varepsilon_{DFP1}) = \sum_{i=1}^{n} \sum_{j=2}^{l} E\left(\frac{\partial D}{\partial d_{ij}} da_{ij}\right) = \sum_{i=1}^{n} \sum_{j=2}^{l} E\left(\frac{\partial D}{\partial d_{ij}} da_{ij}\right) = E(\varepsilon_{DFP2}). \]

The stricter definition (FSC3), is taken as that being the most appropriate of the ones thus far considered. However, in any of these cases there are two required implications. Specifically,

\[ \text{var } \varepsilon_{D1} < \text{var } \varepsilon_{D2} \quad (1) \]

which is true if and only if:
\[ \text{var}(dSFP1) + \text{var}(dNFPD1) + 2\text{cov}(dSFP1, dNFPD1) < \]
\[ \text{var}(dSFP2) + \text{var}(dNFPD2) + 2\text{cov}(dSFP2, dNFPD2) \]

(Condition A*).

The argument above is that the covariance between financial payback and non-financial development payback for any given public investment project is likely to depend on the specific empirical features of that project (as well as the normative weights implicit in the development evaluation function). It follows that whether \( \text{cov}(dSFP1, dNFPD1) < \text{cov}(dSFP2, dNFPD2) \) is an empirical matter. The same would seem to be true of the relative magnitude of the variances of the developmental paybacks and the financial paybacks in the two cases. Nothing in the definition of a public good would seem to provide obvious guidance in assessing the relative magnitude of these variances.

Also:
\[ \text{var} \epsilon_{F1} > \text{var} \epsilon_{F2} \text{ if and only if } \text{var}(dSFP1) > \text{var}(dSFP2) \]  

(Condition B*).

Once again, the relative magnitude of the variances of the financial paybacks would seem to be a matter about which the definition of a public good would not seem to provide clear guidance. It is certainly possible that (1) and (2) are true for reasonable development evaluation functions under specific empirical conditions, but this would depend on empirical facts concerning the nature of public investments.

It is clear that it is necessary to place more structure on the problem if conclusions are to be derived. Returning to Condition A* above, and taking note of the earlier conclusion that there is no theoretical reason to assume that the covariance (between financial and non-financial development payback) is systematically different across projects with different public good characteristics, it is possible to specialize further to cases in which:
\[ \text{cov}(dSFP1, dNFPD1) = \text{cov}(dSFP2, dNFPD2) \]

If 'comparable' projects are defined as ones for which this 'equal covariance' condition holds, in addition to the previous conditions for likeness of projects (that the expected overall development payback and the expected financial payback of the projects are the same), then the fiscal space conjecture can be further refined to state (FSC4): if P1 has greater public good characteristics than P2 (i.e. \( (\alpha_1, \beta_1) > (\alpha_2, \beta_2) \)) and P1 and P2 are comparable projects, then \( \text{var} \epsilon_{D1} < \text{var} \epsilon_{D2} \) and \( \text{var} \epsilon_{F1} > \text{var} \epsilon_{F2} \) where the subscripts refer to the respective public investment projects.
From Condition A*, for this version of the fiscal space conjecture to be true it is required that:

$$\text{Var}(dSFP1) + \text{Var}(dNFDP1) < \text{Var}(dSFP2) + \text{Var}(dNFDP2),$$

(Condition A**) and, as before (Condition B*), that \(\text{Var}(dSFP1) > \text{Var}(dSFP2).\)

It is now possible to introduce an important feature of public goods, which has been considered to be central in the extensive empirical and theoretical literature on the subject. This is the idea that the financing and the provision of the optimal level of public goods is extremely challenging because of difficulties in eliciting the value that individual agents receive from a public investment that has high public good characteristics. The 'free rider problem' in public good provision has been widely viewed as being very serious.\(^5\) Although there are, in principle, mechanisms for eliciting the required information, these are only available under very stringent theoretical conditions. In practice, in real situations, there are considerable uncertainties as to whether the optimal level of the public good can be provided and, more especially, as to whether the resources expended on public good provision can be recovered from those who benefit from them (which will directly affect the governmental financial payback and may indirectly affect the societal financial payback). Such difficulties in recovery may be important not only in creating obstacles to making the public investment initially effective, but in making it difficult to sustain the public investment over time and to generate an adequate ongoing stream of benefits. It might be argued from this fact that it is reasonable to characterize the variance of the financial payback expected from an investment with high public good characteristics as being greater than the variance of the financial payback expected from an investment with low public good characteristics. If this were true, then Condition B* would hold:

$$\text{Var}(dSFP1) > \text{Var}(dSFP2)$$

For Condition A** to hold it is necessary to require in addition that:

$$\text{Var}(dNFDP1) < \text{Var}(dNFDP2)$$

and that the difference in the predictability of the non-financial development payback of the two projects, \(\text{Var}(dNFDP2) - \text{Var}(dNFDP1)\), be sufficiently large. In other words, projects with high public good characteristics must have relatively predictable non-financial returns as compared to projects with low public good characteristics. This assumption may be deemed not unreasonable if it is thought that certain public investment projects possessing high public good characteristics (for instance, public sanitation or immunization campaigns, or rural roads) are known to have fairly
predictable beneficial effects on non-financial development objectives. In contrast, public investment projects possessing lower public good characteristics, such as toll bridges, contribute much less predictably to non-financial development objectives, because the causal interlinkages between the provision of such goods and non-financial development objectives are not well understood.

If these empirical hypotheses are correct then the fiscal space conjecture (FSC4) would hold true. Since the hypotheses required for the proposition to hold true are unverified, albeit plausible, it is correct to describe it as a conjecture.

**CONCLUSIONS**

In the interest of greater clarity, an attempt to make a formal statement of the fiscal space conjecture and to identify some empirical conditions under which it would hold true has been undertaken. The potential implications for public investment planning, especially in developing countries, are worth considering. The risk associated with receiving inadequate financial paybacks from projects that possess high public good characteristics may be high. That may not always be sufficient reason to avoid such projects, especially as the risk associated with the overall development payback of such projects is lower where the fiscal space conjecture holds true. A consequence is that a superficially rational, ‘risk-averse’ public investment policy may have considerable developmental costs, as it may give undue significance to the low predictability of a project’s financial payback, which ought to be only one part of the overall assessment of the risks entailed in public investment.

**NOTES**

1 I am most grateful to Francisco Rodríguez for his helpful suggestions.
2 In this respect it is similar to many other economic and social measures, which involve ‘thick ethical concepts’ – see, for example, Putnam (2004).
3 This is not a restrictive characterization for most public investment projects, which must be viewed as either small relative to a national economy or possible to be disaggregated into small component projects. Moreover, the characterization is convenient but not crucial to the analysis that follows.
4 Excluded is ‘jointness in supply’, mentioned by Roy et al (Chapter 2, this volume), in keeping with current definitions of public goods.
5 See, for example, Cornez and Sandler (1995) for a survey. For an interesting view of the history of this discussion, see Tuck (2008).

**REFERENCES**