2.6 Exercises

Vocabulary Check

Fill in the blanks.

1. Functions of the form $f(x) = N(x)/D(x)$, where $N(x)$ and $D(x)$ are polynomials and $D(x)$ is not the zero polynomial, are called ________ .
2. If $f(x) \to \pm\infty$ as $x \to a$ from the left (or right), then $x = a$ is a ________ of the graph of f.
3. If $f(x) \to b$ as $x \to \pm\infty$, then $y = b$ is a ________ of the graph of f.

In Exercises 1–6, (a) complete each table, (b) determine the vertical and horizontal asymptotes of the function, and (c) find the domain of the function.

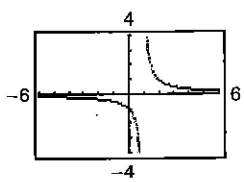
x	$f(x)$
0.5	
0.9	
0.99	
0.999	

x	$f(x)$
1.5	
1.1	
1.01	
1.001	

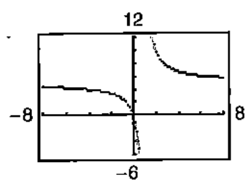
x	$f(x)$
5	
10	
100	
1000	

x	$f(x)$
-5	
-10	
-100	
-1000	

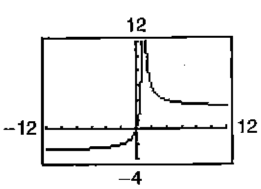
1. $f(x) = \dfrac{1}{x-1}$



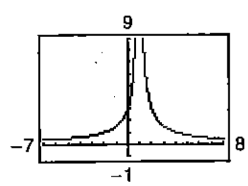
2. $f(x) = \dfrac{5x}{x-1}$



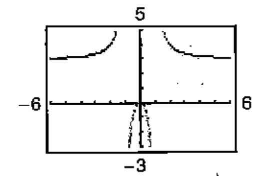
3. $f(x) = \dfrac{3x}{|x-1|}$



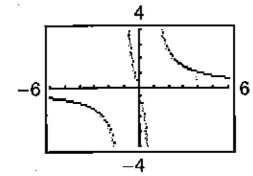
4. $f(x) = \dfrac{3}{|x-1|}$



5. $f(x) = \dfrac{3x^2}{x^2-1}$

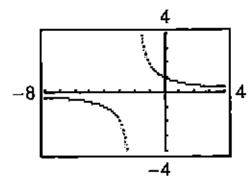


6. $f(x) = \dfrac{4x}{x^2-1}$

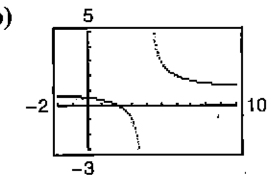


In Exercises 7–12, match the function with its graph. [The graphs are labeled (a), (b), (c), (d), (e), and (f).]

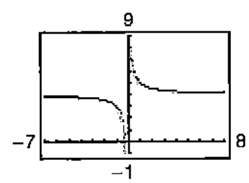
(a)



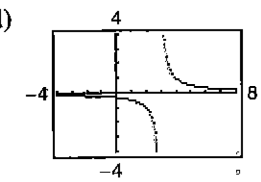
(b)



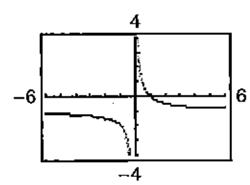
(c)



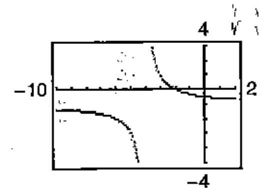
(d)



(e)



(f)



7. $f(x) = \dfrac{2}{x+2}$
8. $f(x) = \dfrac{1}{x-3}$
9. $f(x) = \dfrac{4x+1}{x}$
10. $f(x) = \dfrac{1-x}{x}$
11. $f(x) = \dfrac{x-2}{x-4}$
12. $f(x) = -\dfrac{x+2}{x+4}$

In Exercises 13–22, (a) find the domain of the function, (b) identify any horizontal and vertical asymptotes, and (c) verify your answer to part (a) both graphically by using a graphing utility and numerically by creating a table of values.

13. $f(x) = \frac{1}{x^2}$

14. $f(x) = \frac{3}{(x-2)^3}$

15. $f(x) = \frac{2+x}{2-x}$

16. $f(x) = \frac{1-5x}{1+2x}$

17. $f(x) = \frac{x^2+2x}{2x^2-x}$

18. $f(x) = \frac{x^2-25}{x^2+5x}$

19. $f(x) = \frac{3x^2+x-5}{x^2+1}$

20. $f(x) = \frac{3x^2+1}{x^2+x+9}$

21. $f(x) = \frac{x-3}{|x|}$

22. $f(x) = \frac{x+1}{|x|+1}$

Analytical and Numerical Explanation **In Exercises 23–26, (a) determine the domains of f and g, (b) simplify f and find any vertical asymptotes of f, (c) complete the table, and (d) explain how the two functions differ.**

23. $f(x) = \frac{x^2-4}{x+2}$, $g(x) = x - 2$

x	-4	-3	-2.5	-2	-1.5	-1	0
$f(x)$							
$g(x)$							

24. $f(x) = \frac{x^2(x-3)}{x^2-3x}$, $g(x) = x$

x	-1	0	1	2	3	3.5	4
$f(x)$							
$g(x)$							

25. $f(x) = \frac{x-3}{x^2-3x}$, $g(x) = \frac{1}{x}$

x	-1	-0.5	0	0.5	2	3	4
$f(x)$							
$g(x)$							

26. $f(x) = \frac{2x-8}{x^2-9x+20}$, $g(x) = \frac{2}{x-5}$

x	0	1	2	3	4	5	6
$f(x)$							
$g(x)$							

Exploration **In Exercises 27–30, determine the value that the function f approaches as the magnitude of x increases. Is $f(x)$ greater than or less than this functional value when x is positive and large in magnitude? What about when x is negative and large in magnitude?**

27. $f(x) = 4 - \frac{1}{x}$

28. $f(x) = 2 + \frac{1}{x-3}$

29. $f(x) = \frac{2x-1}{x-3}$

30. $f(x) = \frac{2x-1}{x^2+1}$

In Exercises 31–34, find the zeros (if any) of the rational function. Use a graphing utility to verify your answer.

31. $g(x) = \frac{x^2-4}{x+3}$

32. $g(x) = \frac{x^3-8}{x^2+4}$

33. $f(x) = 1 - \frac{2}{x-5}$

34. $h(x) = 5 + \frac{3}{x^2+1}$

35. ***Environment*** The cost C (in millions of dollars) of removing $p\%$ of the industrial and municipal pollutants discharged into a river is given by

$$C = \frac{255p}{100-p}, \quad 0 \le p < 100.$$

(a) Find the cost of removing 10% of the pollutants.

(b) Find the cost of removing 40% of the pollutants.

(c) Find the cost of removing 75% of the pollutants.

(d) Use a graphing utility to graph the cost function. Be sure to choose an appropriate viewing window. Explain why you chose the values that you used in your viewing window.

(e) According to this model, would it be possible to remove 100% of the pollutants? Explain.

36. ***Environment*** In a pilot project, a rural township is given recycling bins for separating and storing recyclable products. The cost C (in dollars) for supplying bins to $p\%$ of the population is given by

$$C = \frac{25{,}000p}{100 - p}, \quad 0 \le p < 100.$$

(a) Find the cost of supplying bins to 15% of the population.

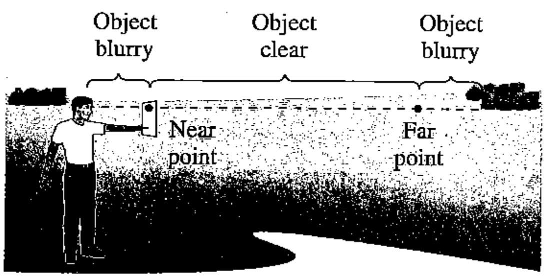
(b) Find the cost of supplying bins to 50% of the population.

(c) Find the cost of supplying bins to 90% of the population.

(d) Use a graphing utility to graph the cost function. Be sure to choose an appropriate viewing window. Explain why you chose the values that you used in your viewing window.

(e) According to this model, would it be possible to supply bins to 100% of the residents? Explain.

37. ***Data Analysis*** The endpoints of the interval over which distinct vision is possible are called the *near point* and *far point* of the eye (see figure). With increasing age these points normally change. The table shows the approximate near points y (in inches) for various ages x (in years).



Age, x	Near point, y
16	3.0
32	4.7
44	9.8
50	19.7
60	39.4

(a) Find a rational model for the data. Take the reciprocals of the near points to generate the points $(x, 1/y)$. Use the *regression* feature of a graphing utility to find a linear model for the data. The resulting line has the form

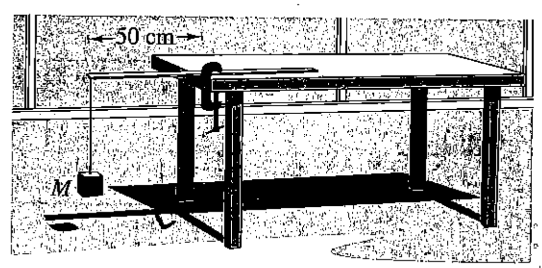
$$\frac{1}{y} = ax + b.$$

Solve for y.

(b) Use the *table* feature of a graphing utility to create a table showing the predicted near point based on the model for each of the ages in the original table.

(c) Do you think the model can be used to predict the near point for a person who is 70 years old? Explain.

38. ***Data Analysis*** Consider a physics laboratory experiment designed to determine an unknown mass. A flexible metal meter stick is clamped to a table with 50 centimeters overhanging the edge (see figure). Known masses M ranging from 200 grams to 2000 grams are attached to the end of the meter stick. For each mass, the meter stick is displaced vertically and then allowed to oscillate. The average time t (in seconds) of one oscillation for each mass is recorded in the table.



Mass, M	Time, t
200	0.450
400	0.597
600	0.721
800	0.831
1000	0.906
1200	1.003
1400	1.088
1600	1.168
1800	1.218
2000	1.338

A model for the data is given by

$$t = \frac{38M + 16{,}965}{10(M + 5000)}.$$

(a) Use the *table* feature of a graphing utility to create a table showing the estimated time based on the model for each of the masses shown in the table. What can you conclude?

(b) Use the model to approximate the mass of an object when the average time for one oscillation is 1.056 seconds.

39. ***Wildlife*** The game commission introduces 100 deer into newly acquired state game lands. The population N of the herd is given by

$$N = \frac{20(5 + 3t)}{1 + 0.04t}, \quad t \geq 0$$

where t is the time in years.

(a) Use a graphing utility to graph the model.

(b) Find the population when $t = 5$, $t = 10$, and $t = 25$.

(c) What is the limiting size of the herd as time increases? Explain.

40. ***Wildlife*** The table shows the number N of threatened and endangered species in the United States from 1993 to 2002. The data can be approximated by the model

$$N = \frac{42.58t^2 + 690}{0.03t^2 + 1}$$

where t represents the year, with $t = 3$ corresponding to 1993. (Source: U.S. Fish and Wildlife Service)

Year	Number, N
1993	813
1994	941
1995	962
1996	1053
1997	1132
1998	1194
1999	1205
2000	1244
2001	1254
2002	1262

(a) Use a graphing utility to plot the data and graph the model in the same viewing window. How closely does the model represent the data?

(b) Use the model to estimate the number of threatened and endangered species in 2006.

(c) Would this model be useful for estimating the number of threatened and endangered species in future years? Explain.

Synthesis

True or False? **In Exercises 41 and 42, determine whether the statement is true or false. Justify your answer.**

41. A rational function can have infinitely many vertical asymptotes.

42. $f(x) = x^3 - 2x^2 - 5x + 6$ is a rational function.

Think About It **In Exercises 43–46, write a rational function f having the specified characteristics. (There are many correct answers.)**

43. Vertical asymptotes: $x = -2, x = 1$

44. Vertical asymptote: None
Horizontal asymptote: $y = 0$

45. Vertical asymptote: None
Horizontal asymptote: $y = 2$

46. Vertical asymptotes: $x = 0, x = \frac{5}{2}$
Horizontal asymptote: $y = -3$

Review

In Exercises 47–50, write the general form of the equation of the line that passes through the points.

47. $(3, 2), (0, -1)$ **48.** $(-6, 1), (4, -5)$

49. $(2, 7), (3, 10)$ **50.** $(0, 0), (-9, 4)$

In Exercises 52–54, divide using long division.

51. $(x^2 + 5x + 6) \div (x - 4)$

52. $(x^2 - 10x + 15) \div (x - 3)$

53. $(2x^2 + x - 11) \div (x + 5)$

54. $(4x^2 + 3x - 10) \div (x + 6)$