2.7 Exercises

Vocabulary Check

Fill in the blanks.

1. For the rational function $f(x) = N(x)/D(x)$, if the degree of $N(x)$ is exactly one more than the degree of $D(x)$, then the graph of f has a _______ (or oblique) _______ .
2. The graph of $f(x) = 1/x$ has a _______ asymptote at $x = 0$.

In Exercises 1–4, use a graphing utility to graph $f(x) = 2/x$ and the function g in the same viewing window. Describe the relationship between the two graphs.

1. $g(x) = f(x) + 1$
2. $g(x) = f(x - 1)$
3. $g(x) = -f(x)$
4. $g(x) = \frac{1}{2}f(x + 2)$

In Exercises 5–8, use a graphing utility to graph $f(x) = 2/x^2$ and the function g in the same viewing window. Describe the relationship between the two graphs.

5. $g(x) = f(x) - 2$
6. $g(x) = -f(x)$
7. $g(x) = f(x - 2)$
8. $g(x) = \frac{1}{4}f(x)$

In Exercises 9–26, sketch the graph of the rational function by hand. As sketching aids, check for intercepts, vertical asymptotes, and horizontal asymptotes. Use a graphing utility to verify your graph.

9. $f(x) = \dfrac{1}{x + 2}$
10. $f(x) = \dfrac{1}{x - 6}$
11. $C(x) = \dfrac{5 + 2x}{1 + x}$
12. $P(x) = \dfrac{1 - 3x}{1 - x}$
13. $f(t) = \dfrac{1 - 2t}{t}$
14. $g(x) = \dfrac{1}{x + 2} + 2$
15. $f(x) = \dfrac{x^2}{x^2 - 4}$
16. $g(x) = \dfrac{x}{x^2 - 9}$
17. $f(x) = \dfrac{x}{x^2 - 1}$
18. $f(x) = -\dfrac{1}{(x - 2)^2}$
19. $g(x) = \dfrac{4(x + 1)}{x(x - 4)}$
20. $h(x) = \dfrac{2}{x^2(x - 3)}$
21. $f(x) = \dfrac{3x}{x^2 - x - 2}$
22. $f(x) = \dfrac{2x}{x^2 + x - 2}$
23. $f(x) = \dfrac{x^2 + 3x}{x^2 + x - 6}$
24. $g(x) = \dfrac{5(x + 4)}{x^2 + x - 12}$
25. $f(x) = \dfrac{x^2 - 1}{x + 1}$
26. $f(x) = \dfrac{x^2 - 16}{x - 4}$

In Exercises 27–36, use a graphing utility to graph the function. Determine its domain and identify any vertical or horizontal asymptotes.

27. $f(x) = \dfrac{2 + x}{1 - x}$
28. $f(x) = \dfrac{3 - x}{2 - x}$
29. $f(t) = \dfrac{3t + 1}{t}$
30. $h(x) = \dfrac{x - 2}{x - 3}$
31. $h(t) = \dfrac{4}{t^2 + 1}$
32. $g(x) = -\dfrac{x}{(x - 2)^2}$
33. $f(x) = \dfrac{x + 1}{x^2 - x - 6}$
34. $f(x) = \dfrac{x + 4}{x^2 + x - 6}$
35. $f(x) = \dfrac{20x}{x^2 + 1} - \dfrac{1}{x}$
36. $f(x) = 5\left(\dfrac{1}{x - 4} - \dfrac{1}{x + 2}\right)$

Exploration **In Exercises 37–42, use a graphing utility to graph the function. What do you observe about its asymptotes?**

37. $h(x) = \dfrac{6x}{\sqrt{x^2 + 1}}$
38. $f(x) = -\dfrac{x}{\sqrt{9 + x^2}}$
39. $g(x) = \dfrac{4|x - 2|}{x + 1}$
40. $f(x) = -\dfrac{8|3 + x|}{x - 2}$
41. $f(x) = \dfrac{4(x - 1)^2}{x^2 - 4x + 5}$
42. $g(x) = \dfrac{3x^4 - 5x + 3}{x^4 + 1}$

In Exercises 43–50, sketch the graph of the rational function by hand. As sketching aids, check for intercepts, vertical asymptotes, and slant asymptotes.

43. $f(x) = \dfrac{2x^2 + 1}{x}$
44. $g(x) = \dfrac{1 - x^2}{x}$

45. $h(x) = \dfrac{x^2}{x - 1}$

46. $f(x) = \dfrac{x^3}{x^2 - 1}$

47. $g(x) = \dfrac{x^3}{2x^2 - 8}$

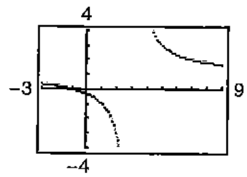
48. $f(x) = \dfrac{x^2 - 1}{x^2 + 4}$

49. $f(x) = \dfrac{x^3 + 2x^2 + 4}{2x^2 + 1}$

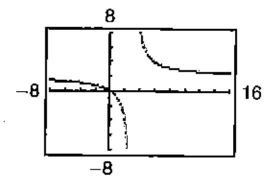
50. $f(x) = \dfrac{2x^2 - 5x + 5}{x - 2}$

Graphical Reasoning **In Exercises 51–54, (a) use the graph to estimate any x-intercepts of the rational function and (b) set $y = 0$ and solve the resulting equation to confirm your result in part (a).**

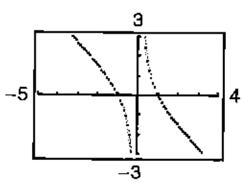
51. $y = \dfrac{x + 1}{x - 3}$



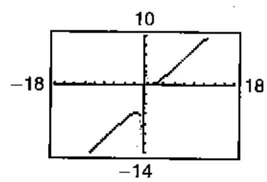
52. $y = \dfrac{2x}{x - 3}$



53. $y = \dfrac{1}{x} - x$



54. $y = x - 3 + \dfrac{2}{x}$



In Exercises 55–58, use a graphing utility to graph the rational function. Determine the domain of the function and identify any asymptotes.

55. $y = \dfrac{2x^2 + x}{x + 1}$

56. $y = \dfrac{x^2 + 5x + 8}{x + 3}$

57. $y = \dfrac{1 + 3x^2 - x^3}{x^2}$

58. $y = \dfrac{12 - 2x - x^2}{2(4 + x)}$

Graphical Reasoning **In Exercises 59–62, (a) use a graphing utility to graph the function and determine any x-intercepts, and (b) set $y = 0$ and solve the resulting equation to confirm your result in part (a).**

59. $y = \dfrac{1}{x + 5} + \dfrac{4}{x}$

60. $y = 20\left(\dfrac{2}{x + 1} - \dfrac{3}{x}\right)$

61. $y = x - \dfrac{6}{x - 1}$

62. $y = x - \dfrac{9}{x}$

63. ***Concentration of a Mixture*** A 1000-liter tank contains 50 liters of a 25% brine solution. You add x liters of a 75% brine solution to the tank.

(a) Show that the concentration C, the proportion of brine to the total solution, of the final mixture is given by

$$C = \frac{3x + 50}{4(x + 50)}.$$

(b) Determine the domain of the function based on the physical constraints of the problem.

(c) Use a graphing utility to graph the function. As the tank is filled, what happens to the rate at which the concentration of brine increases? What percent does the concentration of brine appear to approach?

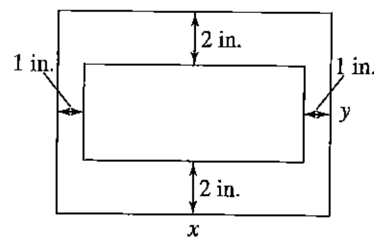
64. ***Geometry*** A rectangular region of length x and width y has an area of 500 square meters.

(a) Write the width y as a function of x.

(b) Determine the domain of the function based on the physical constraints of the problem.

(c) Sketch a graph of the function and determine the width of the rectangle when $x = 30$ meters.

65. ***Page Design*** A page that is x inches wide and y inches high contains 30 square inches of print. The margins at the top and bottom are 2 inches deep and the margins on each side are 1 inch wide (see figure).



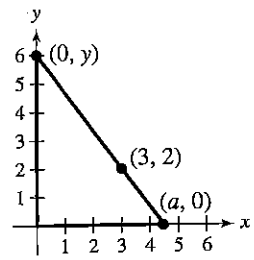
(a) Show that the total area A of the page is given by

$$A = \frac{2x(2x + 11)}{x - 2}.$$

(b) Determine the domain of the function based on the physical constraints of the problem.

(c) Use a graphing utility to graph the area function and approximate the page size such that the minimum amount of paper will be used. Verify your answer numerically using the *table* feature of a graphing utility.

66. ***Geometry*** A right triangle is formed in the first quadrant by the x-axis, the y-axis, and a line segment through the point $(3, 2)$ (see figure).



(a) Show that an equation of the line segment is given by

$$y = \frac{2(a - x)}{a - 3}, \quad 0 \le x \le a.$$

(b) Show that the area of the triangle is given by

$$A = \frac{a^2}{a - 3}.$$

(c) Use a graphing utility to graph the area function and estimate the value of a that yields a minimum area. Estimate the minimum area. Verify your answer numerically using the *table* feature of a graphing utility.

67. ***Cost*** The ordering and transportation cost C (in thousands of dollars) for the components used in manufacturing a product is given by

$$C = 100\left(\frac{200}{x^2} + \frac{x}{x + 30}\right), \quad x \ge 1$$

where x is the order size (in hundreds). Use a graphing utility to graph the cost function. From the graph, estimate the order size that minimizes cost.

68. ***Average Cost*** The cost C of producing x units of a product is given by $C = 0.2x^2 + 10x + 5$, and the average cost per unit is given by

$$\overline{C} = \frac{C}{x} = \frac{0.2x^2 + 10x + 5}{x}, \quad x > 0.$$

Sketch the graph of the average cost function, and estimate the number of units that should be produced to minimize the average cost per unit.

69. ***Medicine*** The concentration C of a chemical in the bloodstream t hours after injection into muscle tissue is given by

$$C = \frac{3t^2 + t}{t^3 + 50}, \quad t \ge 0.$$

(a) Determine the horizontal asymptote of the function and interpret its meaning in the context of the problem.

(b) Use a graphing utility to graph the function and approximate the time when the bloodstream concentration is greatest.

(c) Use a graphing utility to determine when the concentration is less than 0.345.

70. ***Numerical and Graphical Analysis*** A driver averaged 50 miles per hour on the round trip between Baltimore, Maryland and Philadelphia, Pennsylvania, 100 miles away. The average speeds for going and returning were x and miles per hour, respectively.

(a) Show that $y = (25x)/(x - 25)$.

(b) Determine the vertical and horizontal asymptotes of the function.

(c) Use a graphing utility to complete the table. What do you observe?

x	30	35	40	45	50	55	60
y							

(d) Use a graphing utility to graph the function.

(e) Is it possible to average 20 miles per hour in one direction and still average 50 miles per hour on the round trip? Explain.

71. ***Comparing Models*** The attendance A (in millions) at women's Division I college basketball games from 1995 to 2002 is shown in the table. (Source: NCAA)

Year	Attendance, A
1995	4.0
1996	4.2
1997	4.9
1998	5.4
1999	5.8
2000	6.4
2001	6.5
2002	6.9

For each of the following, let t represent the year, with $t = 5$ corresponding to 1995.

(a) Use the *regression* feature of a graphing utility to find a linear model for the data. Use a graphing utility to plot the data and graph the model in the same viewing window.

(b) Find a rational model for the data. Take the reciprocal of A to generate the points $(t, 1/A)$. Use the *regression* feature of a graphing utility to find a linear model for this data. The resulting line has the form

$$\frac{1}{A} = at + b.$$

Solve for A. Use a graphing utility to plot the data and graph the rational model in the same viewing window.

(c) Use the *table* feature of a graphing utility to create a table showing the predicted attendance based on each model for each of the years in the original table. Which model do you prefer? Why?

72. ***Comparing Models*** The table shows the average room rate R (in dollars) for hotels in the United States from 1995 to 2001. The data can be approximated by the model

$$R = \frac{6.245t + 44.05}{0.025t + 1.00}, \quad 5 \le t \le 11$$

where t represents the year, with $t = 5$ corresponding to 1995. (Source: American Hotel & Lodging Association)

Year	Rate, R
1995	66.65
1996	70.93
1997	75.31
1998	78.62
1999	81.33
2000	85.89
2001	88.27

(a) Use a graphing utility to plot the data and graph the model in the same viewing window.

(b) Use the *regression* feature of a graphing utility to find a linear model for the data. Then use a graphing utility to plot the data and graph the linear model in the same viewing window.

(c) Which of the two models would you recommend as a predictor of the average room rate for a hotel for the years following 2001? Explain your reasoning.

Synthesis

True or False? **In Exercises 73 and 74, determine whether the statement is true or false. Justify your answer.**

73. If the graph of a rational function f has a vertical asymptote at $x = 5$, it is possible to sketch the graph without lifting your pencil from the paper.

74. The graph of a rational function can never cross one of its asymptotes.

Think About It **In Exercises 75 and 76, use a graphing utility to graph the function. Explain why there is no vertical asymptote when a superficial examination of the function might indicate that there should be one.**

75. $h(x) = \dfrac{6 - 2x}{3 - x}$ **76.** $g(x) = \dfrac{x^2 + x - 2}{x - 1}$

Think About It **In Exercises 77 and 78, write a rational function satisfying the following criteria.**

77. Vertical asymptote: $x = 2$
Slant asymptote: $y = x + 1$
Zero of the function: $x = -2$

78. Vertical asymptote: $x = -4$
Slant asymptote: $y = x - 2$
Zero of the function: $x = 3$

Review

In Exercises 79–84, simplify the expression.

79. $\left(\dfrac{x}{8}\right)^{-3}$ **80.** $(4x^2)^{-2}$

81. $\dfrac{3x^3y^2}{15xy^4}$ **82.** $\dfrac{(4x^2)^{3/2}}{8x^5}$

83. $\dfrac{3^{7/6}}{3^{1/6}}$ **84.** $\dfrac{x^{-2} \cdot x^{1/2}}{x^{-1} \cdot x^{5/2}}$

In Exercises 85–88, use a graphing utility to graph the function and find its domain and range.

85. $f(x) = \sqrt{6 + x^2}$ **86.** $f(x) = \sqrt{121 - x^2}$

87. $f(x) = -|x + 9|$ **88.** $f(x) = -x^2 + 9$