Chapter 38 Homework (due 12/12/13)

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Problem 38.1
Light of wavelength 540 nm passes through a slit of width 2 mm. The width of the central maximum screen is 8.1 mm.
(a) How far is the screen from the slit?
(b) Determine the width of the first bright fringe to the side of the central maximum.

Solution
(a) Here is the situation.

\[ w \sin \theta = m \lambda \quad \text{where} \quad m = 1, 2, 3, \ldots \]

Using the small angle approximation,
\[ \sin \theta \cong \tan \theta = \frac{y}{L} \]

The first minimum occurs at \( m = 1 \).
\[ \frac{w y}{L} = \lambda \quad \Rightarrow \quad L = \frac{w y}{\lambda} = \frac{(2 \, \text{mm})(4.05 \, \text{mm})}{540 \, \text{nm}} \cdot \frac{10^6 \, \text{nm}}{1 \, \text{mm}} = 1.5 \times 10^4 \, \text{mm} = 15 \, \text{m} \]

(b) The width of the first bright fringe is the distance between the first minimum and the second minimum. The first minimum is at 4.05 mm. The second minimum is at
\[ \frac{w y}{L} = \lambda \quad \Rightarrow \quad y = 2 \frac{L \lambda}{w} = 2 \left( \frac{1.5 \times 10^4 \, \text{mm}}{2 \, \text{mm}} \right) \frac{(540 \, \text{nm})}{10^6 \, \text{nm}} = 8.10 \, \text{mm} \]

The difference is 4.05 mm.
Problem 38.13
The angular resolution of a radio telescope is to be 0.1° when the incident waves have a wavelength of 3 mm. What minimum diameter is required for the telescope’s receiving dish?

Solution
Rayleigh’s criterion says this.

\[
\theta = 1.22 \frac{\lambda}{D} \quad \Rightarrow \quad D = \frac{1.22 \lambda}{\theta} = 1.22 \frac{3 \text{ mm}}{0.1^\circ} \frac{180^\circ}{\pi \text{ rad}} = 2.0970 \text{ m}
\]
Problem 38.17
What is the approximate size of the smallest object on the earth that astronauts can resolve by eye when they are orbiting 250 km above the earth? Assume the wavelength is 500 nm a pupil diameter of 5 mm.

Solution
The angular resolution of the eye is

\[ \theta = 1.22 \frac{\lambda}{D} = 1.22 \frac{500 \text{ nm}}{5 \text{ mm}} \frac{1 \text{ mm}}{10^6 \text{ nm}} = 1.22 \times 10^{-4} \text{ rad} \]

With this angular resolution, the separation between sources \( y \) can be

\[ \tan \theta \cong \theta = \frac{y}{L} \quad \implies \quad y = L\theta = (250 \text{ km})(1.22 \times 10^{-4} \text{ rad}) = 3.05 \times 10^{-2} \text{ km} = 30.5 \text{ m} \]
Problem 38.22

A circular radar antenna on a Coast Guard ship has a diameter of 2.1 m and radiates at a frequency of 15 GHz. Two small boats are located 9 km away from the ship. How close together could the boats and still be detected as two objects?

Solution

The wavelength of the radar signal is

\[
\lambda = \frac{c}{f} = \frac{3 \times 10^8 \text{ m/s}}{15 \times 10^9 \text{ Hz}} = 2 \times 10^{-2} \text{ m}
\]

The angular resolution of the radar is

\[
\theta = 1.22 \frac{\lambda}{D} = 1.22 \frac{2 \times 10^{-2} \text{ m}}{2.1 \text{ m}} = 1.1619 \times 10^{-2} \text{ rad}
\]

With this angular resolution, the separation between sources \(y\) can be

\[
tan(\theta) \cong \theta = \frac{y}{L} \implies y = L \theta = (9 \text{ km})(1.1619 \times 10^{-2} \text{ rad}) = 0.10457 \text{ km} = 104.57 \text{ m}
\]
Problem 38.26
Three discrete spectral lines occur at angles of 10.1°, 13.7°, and 14.8° in the first order spectrum of a grading spectrometer.
(a) If the grating has 3660 slits per centimeter, what are the wavelengths of the light?
(b) At what angles are these lines found in the second-order spectrum?

Solution
(a) The condition for maxima for a diffraction grating is
\[ d \sin \theta = m \lambda \] where \( m = 0,1,2,... \)
The slit separation is
\[ d = \frac{1 \text{ cm}}{3660 \text{ slits}} = 2.7322 \times 10^{-4} \text{ cm} \]
At 10.1°, the wavelength is
\[ (2.7322 \times 10^{-4} \text{ cm}) \sin 10.1° = \lambda \Rightarrow \lambda = 4.7914 \times 10^{-5} \text{ cm} = 479 \text{ nm} \]
At 13.7°, the wavelength is
\[ (2.7322 \times 10^{-4} \text{ cm}) \sin 13.7° = \lambda \Rightarrow \lambda = 6.4710 \times 10^{-5} \text{ cm} = 647 \text{ nm} \]
At 14.8°, the wavelength is
\[ (2.7322 \times 10^{-4} \text{ cm}) \sin 14.8° = \lambda \Rightarrow \lambda = 6.9794 \times 10^{-5} \text{ cm} = 698 \text{ nm} \]
(b) The angles of the second order maxima are, for 479 nm,
\[ (2.7322 \times 10^{-4} \text{ cm}) \sin \theta = 2 \times 4.7914 \times 10^{-5} \text{ cm} \Rightarrow \theta = 20.5° \]
For 647 nm,
\[ (2.7322 \times 10^{-4} \text{ cm}) \sin \theta = 2 \times 6.4710 \times 10^{-5} \text{ cm} \Rightarrow \theta = 28.3° \]
For 698 nm,
\[ (2.7322 \times 10^{-4} \text{ cm}) \sin \theta = 2 \times 6.9794 \times 10^{-5} \text{ cm} \Rightarrow \theta = 30.7° \]
Problem 38.29

A diffraction grating has 4200 rulings per cm. On a screen 2 m from the grating, it is found that for a particular order m, the maxima correspondent to two closely spaced wavelengths of sodium (589.0 nm and 589.6 nm) are separated by 1.54 mm. Determine the value of m.

Solution

The condition for maxima for a diffraction grating is

\[ d \sin \theta = m \lambda \]  
where \( m = 0,1,2,\ldots \)

The slit separation is

\[ d = \frac{1 \text{ cm}}{4200 \text{ slits}} = 2.3810 \times 10^{-4} \text{ cm} = 2381 \text{ nm} \]

The sine of each angle is given by this.

\[ \sin \theta = \frac{y}{\sqrt{(2m)^2 + y^2}} = \frac{y}{\sqrt{4 \times 10^{18} \text{ nm}^2 + y^2}} \]

The position of the maximum is

\[ d \frac{y}{\sqrt{4 \times 10^{18} + y^2}} = m \lambda \quad \Rightarrow \quad d^2 y^2 = m^2 \lambda^2 (4 \times 10^{18} + y^2) \quad \Rightarrow \quad d^2 y^2 - m^2 \lambda^2 y^2 = 4 \times 10^{18} m^2 \lambda^2 \]

\[ y^2 = \frac{4 \times 10^{18} m^2 \lambda^2}{d^2 - m^2 \lambda^2} \quad \Rightarrow \quad y = \frac{2 \times 10^9 m \lambda}{\sqrt{d^2 - m^2 \lambda^2}} \]

The difference in the position of the maxima is

\[ y_2 - y_1 = \frac{2 \times 10^9 m \lambda_2}{\sqrt{d^2 - m^2 \lambda_2^2}} - \frac{2 \times 10^9 m \lambda_1}{\sqrt{d^2 - m^2 \lambda_1^2}} \]

\[ \frac{m(1179.2 \times 10^9)}{\sqrt{5.6692 \times 10^6 - m^2 (3.4763 \times 10^5)}} - \frac{m(1178.0 \times 10^9)}{\sqrt{5.6692 \times 10^6 - m^2 (3.4692 \times 10^5)}} = 1.54 \times 10^6 \]

\[ \frac{m(1179.2 \times 10^3)}{\sqrt{5.6692 \times 10^6 - m^2 (3.4763 \times 10^5)}} - \frac{m(1178.0 \times 10^3)}{\sqrt{5.6692 \times 10^6 - m^2 (3.4692 \times 10^5)}} - 1.54 = 0 \]

The solution for m is 2.0013 or just 2.
Problem 38.33
Lights of wavelength 500 nm is incident normally on a diffraction grating. If the third order maximum of the diffraction pattern is observed at 32°,
(a) What is the number of rulings per centimeter for the grating?
(b) Determine the total number of primary maxima that can be observed in this situation.

Solution
(a) The condition for the maximum is
\[ d \sin \theta = m \lambda \text{ where } m = 0,1,2,... \]
\[ d \sin 32^\circ = 3(500 \text{ nm}) \Rightarrow d = 2830.6 \text{ nm / slit} \Rightarrow 2.8306 \times 10^{-4} \text{ cm / slit} \Rightarrow 3532.8 \text{ slit / cm} \]

(b) The largest observable order is
\[ (2830.6 \text{ nm})\sin 90^\circ = m_{\text{max}}(500 \text{ nm}) \Rightarrow m_{\text{max}} = 5.6612 \text{ or } 5 \]
Problem 38.41
Unpolarized light passes through to ideal Polaroid sheets. The axis of the first is vertical, the axis of the second is at 30° to the vertical. What fraction of the incident light is transmitted?

Solution
The first process of unpolarized light passing through the vertical polarizer results in half of the original intensity.

\[ I_1 = \frac{1}{2} I_0 \]

The second process of vertically polarized light passing through a 30° polarizer results in this much intensity passing through the second polarizer.

\[ I_2 = I_1 \cos^2 30° = \frac{3}{4} I_1 = \frac{3}{8} I_0 \]
Problem 38.47

You use a sequence of ideal polarizing filters each with its axis making the same angle with the axis of the previous filter to rotate the plane of polarization of a polarize light beam by total of 45°. You wish to have an intensity reduction no larger than 10%.

(a) How many polarizers do you need to achieve your goal?

(b) What is the angle between adjacent polarizers?

Solution

(a) When one polarizer is applied, the final intensity is

\[ I_1 = I_0 \cos^2 45° \]

When two polarizers are applied, the angle and the final intensity are

\[ \theta = \frac{45°}{2} \]

\[ I_1 = I_0 \cos^2 \theta \quad \text{and} \quad I_2 = I_1 \cos^2 \theta = I_0 (\cos^2 \theta)^2 \]

The trend is this for \( n \) polarizers.

\[ \theta = \frac{45°}{n} \]

\[ I_n = I_0 \left( \cos \frac{45°}{n} \right)^{2n} \]

The constrain here is that the coefficient is at most 0.9.

\[ \left( \cos \frac{45°}{n} \right)^{2n} > 0.9 \]

When equal, the value of \( n \) is 5.8722. When using 5 polarizers, the coefficient is 0.88349 which is too much. When using 6 polarizers, the coefficient is 0.90203 which is fine.