7-2 Factoring by GCF

Warm Up

Simplify.

1. $2(w + 1)$ $2w + 2$

2. $3x(x^2 - 4)$ $3x^3 - 12x$

Find the GCF of each pair of monomials.

3. $4h^2$ and $6h$ $2h$

4. $13p$ and $26p^5$ $13p$

7-2 Factoring by GCF

Objective

Factor polynomials by using the greatest common factor.

7-2 Factoring by GCF

Recall that the Distributive Property states that $ab + ac = a(b + c)$. The Distributive Property allows you to "factor" out the GCF of the terms in a polynomial to write a factored form of the polynomial.

A polynomial is in its factored form when it is written as a product of monomials and polynomials that cannot be factored further. The polynomial $2(3x - 4x)$ is not fully factored because the terms in the parentheses have a common factor of x.

Example 1A: Factoring by Using the GCF

Factor each polynomial. Check your answer.

$2x^2 - 4$

$2x^2 = \boxed{2} \cdot x \cdot x$

$4 = \boxed{2} \cdot 2$

Find the GCF.

2

The GCF of $2x^2$ and 4 is 2.

$2x^2 - (2 \cdot 2)$

Write terms as products using the GCF as a factor.

$2(x^2 - 2)$

Use the Distributive Property to factor out the GCF.

Check $2(x^2 - 2)$

Multiply to check your answer.

$2x^2 - 4$ ✓

The product is the original polynomial.

7-2 Factoring by GCF

Writing Math

Aligning common factors can help you find the greatest common factor of two or more terms.

Example 1B: Factoring by Using the GCF

Factor each polynomial. Check your answer.

$8x^3 - 4x^2 - 16x$

$8x^3 = 2 \cdot 2 \cdot 2 \cdot x \cdot x \cdot x$

$4x^2 = 2 \cdot 2 \cdot x \cdot x$

$16x = 2 \cdot 2 \cdot 2 \cdot 2 \cdot x$

$2 \cdot 2 \cdot x = 4x$

Find the GCF.

The GCF of $8x^3$, $4x^2$, and $16x$ is $4x$.

$2x^2(4x) - x(4x) - 4(4x)$

Write terms as products using the GCF as a factor.

$4x(2x^2 - x - 4)$

Use the Distributive Property to factor out the GCF.

Check $4x(2x^2 - x - 4)$

Multiply to check your answer.

$8x^3 - 4x^2 - 16x$ ✓

The product is the original polynomials.

Example 1C: Factoring by Using the GCF

Factor each polynomial. Check your answer.

$-14x - 12x^2$

$-1(14x + 12x^2)$

$14x = 2 \cdot 7 \cdot x$

$12x^2 = 2 \cdot 2 \cdot 3 \cdot x \cdot x$

$2 \cdot x = 2x$

Both coefficients are negative. Factor out –1.

Find the GCF.

The GCF of $14x$ and $12x^2$ is $2x$.

$-1[7(2x) + 6x(2x)]$

Write each term as a product using the GCF.

$-1[2x(7 + 6x)]$

Use the Distributive Property to factor out the GCF.

$-2x(7 + 6x)$

Example 1C: Continued

Factor each polynomial. Check your answer.

$-14x - 12x^2$

Check $-2x(7 + 6x)$ *Multiply to check your answer.*

$-14x - 12x^2$ ✓ *The product is the original polynomial.*

Caution!

When you factor out –1 as the first step, be sure to include it in all the other steps as well.

Example 1D: Factoring by Using the GCF

Factor each polynomial. Check your answer.

$3x^3 + 2x^2 - 10$

$3x^3 = 3 \cdot x \cdot x \cdot x$ *Find the GCF.*

$2x^2 = 2 \cdot x \cdot x$

$10 = 2 \cdot 5$

$3x^3 + 2x^2 - 10$

There are no common factors other than 1.

The polynomial cannot be factored further.

Check It Out! Example 1a

Factor each polynomial. Check your answer.

$5b + 9b^3$

$5b = 5 \cdot \boxed{b}$

$9b = 3 \cdot 3 \cdot \boxed{b} \cdot b \cdot b$

$\boxed{b}$

$5(b) + 9b^2(b)$

$b(5 + 9b^2)$

Check $b(5 + 9b^2)$

$5b + 9b^3$ ✓

Find the GCF.

The GCF of $5b$ and $9b^3$ is b.

Write terms as products using the GCF as a factor.

Use the Distributive Property to factor out the GCF.

Multiply to check your answer.

The product is the original polynomial.

Check It Out! Example 1b

Factor each polynomial. Check your answer.

$9d^2 - 8^2$

$9d^2 = 3 \cdot 3 \cdot d \cdot d$ *Find the GCF.*

$8^2 = 2 \cdot 2 \cdot 2 \cdot 2 \cdot 2 \cdot 2$

$9d^2 - 8^2$ *There are no common factors other than 1.*

The polynomial cannot be factored further.

Check It Out! Example 1c

Factor each polynomial. Check your answer.

$-18y^3 - 7y^2$

$-1(18y^3 + 7y^2)$

Both coefficients are negative. Factor out –1.

$18y^3 = 2 \cdot 3 \cdot 3 \cdot y \cdot y \cdot y$

$7y^2 = 7 \cdot y \cdot y$

Find the GCF.

$y \cdot y = y^2$ *The GCF of $18y^3$ and $7y^2$ is y^2.*

$-1[18y(y^2) + 7(y^2)]$

Write each term as a product using the GCF.

$-1[y^2(18y + 7)]$

Use the Distributive Property to factor out the GCF..

$-y^2(18y + 7)$

Check It Out! Example 1d

Factor each polynomial. Check your answer.

$$8x^4 + 4x^3 - 2x^2$$

$8x^4 = \boxed{2} \cdot 2 \cdot 2 \cdot \boxed{x} \cdot \boxed{x} \cdot x \cdot x$

$4x^3 = \boxed{2} \cdot 2 \cdot x \cdot \boxed{x} \cdot \boxed{x}$

$2x^2 = \boxed{2} \cdot \qquad\quad \boxed{x} \cdot \boxed{x}$

$2 \cdot \qquad\quad x \cdot x = 2x^2$

Find the GCF.

The GCF of $8x^4$, $4x^3$ and $-2x^2$ is $2x^2$.

$4x^2(2x^2) + 2x(2x^2) - 1(2x^2)$

Write terms as products using the GCF as a factor.

$2x^2(4x^2 + 2x - 1)$

Use the Distributive Property to factor out the GCF.

Check $2x^2(4x^2 + 2x - 1)$

Multiply to check your answer.

$8x^4 + 4x^3 - 2x^2$

The product is the original polynomial.

7-2 Factoring by GCF

To write expressions for the length and width of a rectangle with area expressed by a polynomial, you need to write the polynomial as a product. You can write a polynomial as a product by factoring it.

Example 2: *Application*

The area of a court for the game squash is ($9x^2 + 6x$) square meters. Factor this polynomial to find possible expressions for the dimensions of the squash court.

$A = 9x^2 + 6x$ — *The GCF of $9x^2$ and $6x$ is $3x$.*

$= 3x(3x) + 2(3x)$ — *Write each term as a product using the GCF as a factor.*

$= 3x(3x + 2)$ — *Use the Distributive Property to factor out the GCF.*

Possible expressions for the dimensions of the squash court are $3x$ m and $(3x + 2)$ m.

Check It Out! Example 2

What if...? The area of the solar panel on another calculator is $(2x^2 + 4x)$ cm^2. Factor this polynomial to find possible expressions for the dimensions of the solar panel.

$A = 2x^2 + 4x$ — *The GCF of $2x^2$ and $4x$ is $2x$.*

$= x(2x) + 2(2x)$ — *Write each term as a product using the GCF as a factor.*

$= 2x(x + 2)$ — *Use the Distributive Property to factor out the GCF.*

Possible expressions for the dimensions of the solar panel are $2x$ cm, and $(x + 2)$ cm.

7-2 Factoring by GCF

Sometimes the GCF of terms is a binomial. This GCF is called a common binomial factor. You factor out a common binomial factor the same way you factor out a monomial factor.

Example 3: Factoring Out a Common Binomial Factor

Factor each expression.

A. $5(x + 2) + 3x(x + 2)$

$5(x + 2) + 3x(x + 2)$ — *The terms have a common binomial factor of $(x + 2)$.*

$(x + 2)(5 + 3x)$ — *Factor out $(x + 2)$.*

B. $-2b(b^2 + 1) + (b^2 + 1)$

$-2b(b^2 + 1) + (b^2 + 1)$ — *The terms have a common binomial factor of $(b^2 + 1)$.*

$-2b(b^2 + 1) + 1(b^2 + 1)$ — *$(b^2 + 1) = 1(b^2 + 1)$*

$(b^2 + 1)(-2b + 1)$ — *Factor out $(b^2 + 1)$.*

Example 3: Factoring Out a Common Binomial Factor

Factor each expression.

C. $4z(z^2 - 7) + 9(2z^3 + 1)$

$4z(z^2 - 7) + 9(2z^3 + 1)$

There are no common factors.

The expression cannot be factored.

Check It Out! Example 3

Factor each expression.

a. $4s(s + 6) – 5(s + 6)$

$4s(s + 6) – 5(s + 6)$ — *The terms have a common binomial factor of (s + 6).*

$(4s – 5)(s + 6)$ — *Factor out (s + 6).*

b. $7x(2x + 3) + (2x + 3)$

$7x(2x + 3) + (2x + 3)$ — *The terms have a common binomial factor of (2x + 3).*

$7x(2x + 3) + 1(2x + 3)$ — *(2x + 3) = 1(2x + 3)*

$(2x + 3)(7x + 1)$ — *Factor out (2x + 3).*

Check It Out! Example 3 : Continued

Factor each expression.

c. $3x(y + 4) – 2y(x + 4)$

$3x(y + 4) – 2y(x + 4)$

There are no common factors.

The expression cannot be factored.

d. $5x(5x – 2) – 2(5x – 2)$

$5x(5x – 2) – 2(5x – 2)$

$(5x – 2)(5x – 2)$

$(5x – 2)^2$

The terms have a common binomial factor of $(5x – 2)$.

$(5x – 2)(5x – 2) = (5x – 2)^2$

7-2 Factoring by GCF

You may be able to factor a polynomial by grouping. When a polynomial has four terms, you can make two groups and factor out the GCF from each group.

Example 4A: Factoring by Grouping

Factor each polynomial by grouping. Check your answer.

$6h^4 - 4h^3 + 12h - 8$

$(6h^4 - 4h^3) + (12h - 8)$ *Group terms that have a common number or variable as a factor.*

$2h^3(3h - 2) + 4(3h - 2)$ *Factor out the GCF of each group.*

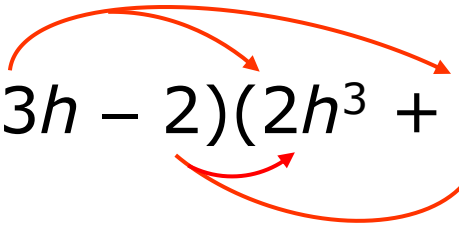
$2h^3(3h - 2) + 4(3h - 2)$ *(3h – 2) is another common factor.*

$(3h - 2)(2h^3 + 4)$ *Factor out (3h – 2).*

Example 4A Continued

Factor each polynomial by grouping. Check your answer.

Check $(3h - 2)(2h^3 + 4)$



Multiply to check your solution.

$3h(2h^3) + 3h(4) - 2(2h^3) - 2(4)$

$6h^4 + 12h - 4h^3 - 8$

$6h^4 - 4h^3 + 12h - 8$ ✓

The product is the original polynomial.

Example 4B: Factoring by Grouping

Factor each polynomial by grouping. Check your answer.

$5y^4 - 15y^3 + y^2 - 3y$

$(5y^4 - 15y^3) + (y^2 - 3y)$ *Group terms.*

$5y^3(y - 3) + y(y - 3)$ *Factor out the GCF of each group.*

$5y^3(y - 3) + y(y - 3)$ *$(y - 3)$ is a common factor.*

$(y - 3)(5y^3 + y)$ *Factor out $(y - 3)$.*

Example 4B Continued

Factor each polynomial by grouping. Check your answer.

$5y^4 - 15y^3 + y^2 - 3y$

Check $(y - 3)(5y^3 + y)$

$y(5y^3) + y(y) - 3(5y^3) - 3(y)$ *Multiply to check your solution.*

$5y^4 + y^2 - 15y^3 - 3y$

$5y^4 - 15y^3 + y^2 - 3y$ ✓ *The product is the original polynomial.*

Check It Out! Example 4a

Factor each polynomial by grouping. Check your answer.

$6b^3 + 8b^2 + 9b + 12$

$(6b^3 + 8b^2) + (9b + 12)$ — *Group terms.*

$2b^2(3b + 4) + 3(3b + 4)$ — *Factor out the GCF of each group.*

$2b^2(3b + 4) + 3(3b + 4)$ — *(3b + 4) is a common factor.*

$(3b + 4)(2b^2 + 3)$ — *Factor out (3b + 4).*

Check It Out! Example 4a Continued

Factor each polynomial by grouping. Check your answer.

$6b^3 + 8b^2 + 9b + 12$

Check $(3b + 4)(2b^2 + 3)$

Multiply to check your solution.

$3b(2b^2) + 3b(3) + (4)(2b^2) + (4)(3)$

$6b^3 + 9b + 8b^2 + 12$

$6b^3 + 8b^2 + 9b + 12$ ✓

The product is the original polynomial.

Check It Out! Example 4b

Factor each polynomial by grouping. Check your answer.

$4r^3 + 24r + r^2 + 6$

$(4r^3 + 24r) + (r^2 + 6)$ — *Group terms.*

$4r(r^2 + 6) + 1(r^2 + 6)$ — *Factor out the GCF of each group.*

$4r(r^2 + 6) + 1(r^2 + 6)$ — *$(r^2 + 6)$ is a common factor.*

$(r^2 + 6)(4r + 1)$ — *Factor out $(r^2 + 6)$.*

Check It Out! Example 4b Continued

Factor each polynomial by grouping. Check your answer.

Check $(4r + 1)(r^2 + 6)$

$4r(r^2) + 4r(6) + 1(r^2) + 1(6)$ *Multiply to check your solution.*

$4r^3 + 24r + r^2 + 6$

$4r^3 + 24r + r^2 + 6$ ✓ *The product is the original polynomial.*

Helpful Hint

If two quantities are opposites, their sum is 0.

$$(5 - x) + (x - 5)$$

$$5 - x + x - 5$$

$$-x + x + 5 - 5$$

$$0 + 0$$

$$0$$

Recognizing opposite binomials can help you factor polynomials. The binomials $(5 - x)$ and $(x - 5)$ are opposites. Notice $(5 - x)$ can be written as $-1(x - 5)$.

$$-1(x - 5) = (-1)(x) + (-1)(-5)$$ *Distributive Property.*

$$= -x + 5$$ *Simplify.*

$$= 5 - x$$ *Commutative Property of Addition.*

So, $(5 - x) = -1(x - 5)$

Example 5: Factoring with Opposites

Factor $2x^3 - 12x^2 + 18 - 3x$ by grouping.

$2x^3 - 12x^2 + 18 - 3x$

$(2x^3 - 12x^2) + (18 - 3x)$ — *Group terms.*

$2x^2(x - 6) + 3(6 - x)$ — *Factor out the GCF of each group.*

$2x^2(x - 6) + 3(-1)(x - 6)$ — *Write $(6 - x)$ as $-1(x - 6)$.*

$2x^2(x - 6) - 3(x - 6)$ — *Simplify. $(x - 6)$ is a common factor.*

$(x - 6)(2x^2 - 3)$ — *Factor out $(x - 6)$.*

Check It Out! Example 5a

Factor each polynomial by grouping.

$15x^2 - 10x^3 + 8x - 12$

$(15x^2 - 10x^3) + (8x - 12)$ *Group terms.*

$5x^2(3 - 2x) + 4(2x - 3)$ *Factor out the GCF of each group.*

$5x^2(3 - 2x) + 4(-1)(3 - 2x)$ *Write $(2x - 3)$ as $-1(3 - 2x)$.*

$5x^2(3 - 2x) - 4(3 - 2x)$ *Simplify. $(3 - 2x)$ is a common factor.*

$(3 - 2x)(5x^2 - 4)$ *Factor out $(3 - 2x)$.*

Check It Out! Example 5b

Factor each polynomial by grouping.

$8y - 8 - x + xy$

$(8y - 8) + (-x + xy)$ *Group terms.*

$8(y - 1) + (x)(-1 + y)$ *Factor out the GCF of each group.*

$8(y - 1) + (x)(y - 1)$ *(y – 1) is a common factor.*

$(y - 1)(8 + x)$ *Factor out (y – 1) .*

Lesson Quiz: Part I

Factor each polynomial. Check your answer.

1. $16x + 20x^3$ $4x(4 + 5x^2)$

2. $4m^4 - 12m^2 + 8m$ $4m(m^3 - 3m + 2)$

Factor each expression.

3. $7k(k - 3) + 4(k - 3)$ $(k - 3)(7k + 4)$

4. $3y(2y + 3) - 5(2y + 3)$ $(2y + 3)(3y - 5)$

Lesson Quiz: Part II

Factor each polynomial by grouping. Check your answer.

5. $2x^3 + x^2 - 6x - 3$ $(2x + 1)(x^2 - 3)$

6. $7p^4 - 2p^3 + 63p - 18$ $(7p - 2)(p^3 + 9)$

7. A rocket is fired vertically into the air at 40 m/s. The expression $-5t^2 + 40t + 20$ gives the rocket's height after t seconds. Factor this expression. $-5(t^2 - 8t - 4)$