

Things you should know from 13.3, 4, 5:

13.3

- Be familiar with the three common notations for partial derivatives. That is, given a function $f(x, y, z)$, we may write a partial derivative in x as $\frac{\partial f}{\partial x}$, f_x , or $\partial_x f$.
- What is the limit definition of a partial derivative? For example, for a function $f(x, y, z)$, how can you define $\frac{\partial f}{\partial x}$ in terms of a limit?
- What does the partial derivative tell us about a function $f(x, y, z)$? Think geometrically. It may be easier to reason about this with a function of two variables, $f(x, y)$.
- What are the common partial derivative rules? See the yellow box on page 836 of your text.
- How, in practice, do you **compute** a partial derivative of some function $f(x, y, z)$?
- What are the mixed partial derivatives of a function $f(x, y)$? How do you compute them?
- If a given function $f(x, y)$ is ‘nice¹,’ what can we say about its mixed partial derivatives? Does this fact hold when $f(x, y)$ is **not** ‘nice?’

13.4

- Really, you just need to understand how to draw a tree-of-derivatives² with the function being differentiated on the left and the dependent variables on the right, and how to traverse that tree to construct the appropriate chain rule. If you understand this, the rest of these pointers are redundant (except maybe the last one). See page 849 of your text if my explanation of the tree-of-derivatives did not make sense. (You are reading your textbook?)
- Given a function $f(x, y, z)$ where we know $x(t), y(t)$, and $z(t)$, how do we compute $\frac{df}{dt}$?
- Given a function $f(x, y, z)$ where we know $x(t, s), y(t, s)$ and $z(t, s)$, how do we compute $\frac{\partial f}{\partial s}$?
- How is the chain rule in this section related to implicit differentiation, a topic you may remember from high school calculus / Calculus I? Hint: for an equation $F(x, y(x)) = G(x, y(x))$, we may define $H(x, y(x)) = F(x, y(x)) - G(x, y(x)) = 0$, compute $\frac{d}{dx}H(x, y(x)) = 0$ and solve for $\frac{dy}{dx}$.

¹Where ‘nice’ here means that the mixed partials f_{xy} and f_{yx} are continuous at the point we wish to compute the mixed partials.

²What I incorrectly called in class a ‘trellis.’ Trellis means something specific in mathematics. See for instance here: [http://en.wikipedia.org/wiki/Trellis_\(graph\)](http://en.wikipedia.org/wiki/Trellis_(graph)).

13.5

- What is the directional derivative of a function $f(x, y, z)$ at a point (x_0, y_0, z_0) in the direction of a unit vector $\mathbf{u} = \langle u_1, u_2, u_3 \rangle$? We call this object $D_{\mathbf{u}}f(x_0, y_0, z_0)$. How do we get this computationally? What does it mean geometrically?
- How are first partial derivatives a special case of directional derivatives? In what directions?
- How is the directional derivative defined in terms of a limit?
- If you are asked to compute the directional derivative of f in the direction \mathbf{a} at the point (x_0, y_0, z_0) , what must you be sure to do? Hint: what if \mathbf{a} isn't a unit vector?