

## Things you should know from 12.4, 5, 6:

### 12.4

- What makes a curve  $C$  traced out by a vector-valued function  $\mathbf{r}(t)$  smooth (two conditions on  $\mathbf{r}'(t)$ )? Piecewise smooth?
- For the purposes of this course, a curve is either smooth or piecewise smooth<sup>1</sup>.
- What makes a curve closed?
- How do you compute the length of a curve  $C$  traced out by a vector-valued function  $\mathbf{r}'(t)$  over a time interval  $a \leq t \leq b$ ?
- How do you compute the arclength of a curve starting from some time point  $t = a$ ?
- What is  $\sqrt{x^2}$ . Hint: the answer is **not**  $x$ .

### 12.5

- What is the tangent vector  $\mathbf{T}(t)$  to some vector-valued function  $\mathbf{r}(t)$ ?
- What is the normal vector  $\mathbf{N}(t)$  to some vector-valued function  $\mathbf{r}(t)$ ? Why is it normal to  $\mathbf{T}(t)$ ?
- What are the magnitudes of  $\mathbf{T}(t)$  and  $\mathbf{N}(t)$ ?
- What do  $\mathbf{T}(t)$  and  $\mathbf{N}(t)$  mean geometrically with respect to the curve  $C$  traced out by a vector-valued function  $\mathbf{r}(t)$ ?
- What is the binormal vector  $\mathbf{B}(t)$ ?
- What are the magnitudes of the components of the acceleration  $\mathbf{a}(t)$  in the direction of  $\mathbf{T}(t)$  ( $a_T$ ) and  $\mathbf{N}(t)$  ( $a_N$ )? There are multiple forms for these. Use whichever is easiest.

### 12.6

- What is the curvature  $\kappa(t)$  of a curve  $C$  traced out by the vector-valued function  $\mathbf{r}(t)$ ? There are two useful forms for this, depending on if you prefer using the quotient rule for derivatives or computing cross products.
- What is the radius of curvature  $\rho(t)$  for some curve  $C$ ? Why does it have 'radius' in its name? (Hint: the answer involves circles).
- What is the curvature of a line  $\ell$ ?

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<sup>1</sup>For the purposes of reality, see the Weierstrass function for an example of a curve that is neither smooth nor piecewise smooth: [http://en.wikipedia.org/wiki/Weierstrass\\_function](http://en.wikipedia.org/wiki/Weierstrass_function). In fact, the Weierstrass curve is everywhere continuous but nowhere differentiable. Weird, right?