Quiz 3

MATH 241 Quiz 3

Answer the questions in the spaces provided. If you run out of room for an answer, continue on the back of the page.

Name: ______________________________

1. An object has the position $\left(-\frac{1}{\pi^2}, 0, -\frac{1}{\pi^2}\right)$ at $t = 1$, velocity $\mathbf{i} + \frac{1}{2}e^2\mathbf{j}$ at $t = 1$, and acceleration $\mathbf{a}(t) = \frac{2}{t^2}\mathbf{i} + e^{2t}\mathbf{j} + \cos(\pi t)\mathbf{k}$. Find the vector valued functions to describe the object's position and velocity.

   Solution: To get from the acceleration to the velocity function, we integrate each component and use the initial condition to determine the appropriate constant of integration.

$$\int \frac{2}{t^2}dt = -\frac{2}{t} + c$$

When $t = 1$ the $\mathbf{i}$ component of velocity is $1 \Rightarrow -2 + c = 1 \Rightarrow c = 3$

$$\int e^{2t}dt = \frac{1}{2}e^{2t} + c$$

When $t = 1$ the $\mathbf{j}$ component of velocity is $\frac{1}{2}e^2 \Rightarrow \dfrac{1}{2}e^2 + c = \dfrac{1}{2}e^2 \Rightarrow c = 0$

$$\int \cos(\pi t)dt = \frac{1}{\pi}\sin(\pi t) + c$$

When $t = 1$ the $\mathbf{k}$ component of velocity is $0 \Rightarrow \dfrac{1}{\pi}\sin(\pi) + c = 0 \Rightarrow c = 0$

The velocity function can then be written:

$$\mathbf{v}(t) = \left(-\frac{2}{t} + 3\right)\mathbf{i} + \frac{1}{2}e^{2t}\mathbf{j} + \frac{1}{\pi}\sin(\pi t)\mathbf{k}$$

The same procedure is followed to get from the velocity function to the position function.

$$\int \left(-\frac{2}{t} + 3\right)dt = -2\ln(t) + 3t + c$$

When $t = 1$ the $\mathbf{i}$ component of position is $-\frac{1}{\pi^2} \Rightarrow 3 + c = -\dfrac{1}{\pi^2} \Rightarrow c = -3 - \dfrac{1}{\pi^2}$

$$\int \frac{1}{2}e^{2t}dt = \frac{1}{4}e^{2t} + c$$

When $t = 1$ the $\mathbf{j}$ component of position is $0 \Rightarrow \dfrac{1}{4}e^2 + c = 0 \Rightarrow c = -\dfrac{1}{4}e^2$

$$\int \left(\frac{1}{\pi}\sin(\pi t)\right)dt = -\frac{1}{\pi^2}\cos(\pi t) + c$$

When $t = 1$ the $\mathbf{k}$ component of position is $-\frac{1}{\pi^2} \Rightarrow \dfrac{1}{\pi^2} + c = -\dfrac{1}{\pi^2} \Rightarrow c = -\dfrac{2}{\pi^2}$

The velocity function can then be written:

$$\mathbf{p}(t) = \left(-2\ln(t) + 3t - 3 - \frac{1}{\pi^2}\right)\mathbf{i} + \left(\frac{1}{4}e^{2t} - \frac{1}{4}e^2\right)\mathbf{j} + \left(-\frac{1}{\pi^2}\cos(\pi t) - \frac{2}{\pi^2}\right)\mathbf{k}$$

2. Compute the limit

$$\lim_{t\to -3} \ln(-t)\mathbf{i} + \frac{t^2+2t-3}{t+3}\mathbf{j} + \frac{\sin(t+3)}{t+3}\mathbf{k}$$

Solution: To compute the limit we examine each component of the vector valued function individually.

The first limit is straight forward and we can just plug in since the function is continuous at the limiting value.

$$\lim_{t\to 3} \ln(-t) = \ln(3)$$

The second limit's numerator is factorable, allowing us to cancel with the denominator and plug in the limit value without issue.

$$\lim_{t\to 3} \frac{t^2+2t-3}{t+3} = \lim_{t\to 3} \frac{(t+3)(t-1)}{t+3} = \lim_{t\to 1}(t-1) = -4$$

For the third limit we see that taking the limit of the numerator and denominator individually gives a $\frac{0}{0}$ in which case we can use L'Hôpital's rule to find the limit.

$$\lim_{t\to 3} \frac{\sin(t+3)}{t+3} = \lim_{t\to 1} \frac{\frac{d}{dt}\sin(t+3)}{\frac{d}{dt}(t+3)} = \lim_{t\to 1} \frac{\cos(t+3)}{1} = 1$$

Combining all of the limits in vector notation produces the final result:

$$\lim_{t\to -3} \ln(-t)\mathbf{i} + \frac{t^2+2t-3}{t+3}\mathbf{j} + \frac{\sin(t+3)}{t+3}\mathbf{k} = \ln(3)\mathbf{i} - 4\mathbf{j} + \mathbf{k}$$