Quiz 2

MATH 241 Quiz 2

Answer the questions in the spaces provided. If you run out of room for an answer, continue on the back of the page.

Name: _____

1. Find a vector perpindicular to both $\vec{a} = (4, -1, 0)$ and $\vec{b} = (2, 1, 3)$.

   Solution: Taking the cross product of two non-parallel vectors is the easiest way to find a third vector which is perpindicular to both $\vec{a}$ and $\vec{b}$:

   $$M = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 4 & -1 & 0 \\ 2 & 1 & 3 \end{vmatrix} = (-3 - 0)\hat{i} - (12 - 0)\hat{j} + (4 + 2)\hat{k}$$

   Thus, one possible solution is $(-3, 12, 6)$.

2. Let $P_1 = (1, 2, -1)$ and $P_2 = (-1, 0, 3)$. Find any (parametric) equation for the line which contains both points.

   Solution: Using the general parametric form for a line, $l(t) = \vec{v_1} + \vec{v_2}t$, we need to select $\vec{v_1}$ to be any point along the line, and $\vec{v_2}$ to be a vector parallel to the line. For the former, any choice such as P_1 is sufficient. For the latter, the simplest choice is the vector $\vec{P_1P_2} = (-2, -2, 4)$. This gives parametric form

   $$l(t) = (1, 2, -1) + (-2t, -2t, 4t) = (1 - 2t)\hat{i} + (1 - 2t)\hat{j} + (1 + 4t)\hat{k}$$

3. The equation for the set of all points which are the same distance from the two points $(3, 2, 3) and (-1, 2, 1)$ is the equation for a plane. What is this equation?

   Solution: First we consider the equations for the distance from an arbitrary point (x, y, z) to either of the two given points:

$$D_1 = \sqrt{(x-3)^2 + (y-2)^2 + (z-3)^2}$$

$$D_2 = \sqrt{(x+1)^2 + (y-2)^2 + (z-1)^2}$$

   Since we want to look at the points which are equidistant, we set $D_1 = D_2$, square both sides, and expand.

$$x^2 - 6x + 9 + y^2 - 4y + 4 + z^2 - 6z + 9 = x^2 + 2x + 1 + y^2 - 4y + 4 + z^2 - 2z + 1$$

   Then cancelling the x^2, y^2, and z^2 terms yields

$$-8x + 0y - 4z + 16 = 0$$