Quiz 1

MATH 241 Quiz 1

Answer the questions in the spaces provided. If you run out of room for an answer, continue on the back of the page.

Name: _____

1. Find a unit vector $\hat{\mathbf{a}}$ having the same direction as the vector $\mathbf{a} = 2\mathbf{i} - \mathbf{j} + 3\mathbf{k}$.

   Solution: We want to find $\hat{\mathbf{a}}$ such that $||\hat{\mathbf{a}}|| = 1$. Note that

   $$\begin{aligned} ||\hat{\mathbf{a}}|| &= \left|\left| \frac{\mathbf{a}}{||\mathbf{a}||} \right|\right| \\ &= \frac{||\mathbf{a}||}{||\mathbf{a}||} \\ &= 1. \end{aligned}$$

   Thus, computing

   $$||\mathbf{a}|| = \sqrt{2^2 + (-1)^2 + 3^2} = \sqrt{14},$$

   we see that $\hat{\mathbf{a}} = \frac{1}{||\mathbf{a}||}\mathbf{a} = \frac{1}{\sqrt{14}}(2\mathbf{i} - \mathbf{j} + 3\mathbf{k})$.

2. Show that the vectors

   $$\mathbf{i} + \mathbf{k}, \mathbf{i} + 2\mathbf{j} - \mathbf{k}, -\mathbf{i} + \mathbf{j} + \mathbf{k}$$

   are mutually perpendicular.

   Solution: Computing all pairwise dot products, we see

   - $(\mathbf{i} + \mathbf{k}) \cdot (\mathbf{i} + 2\mathbf{j} - \mathbf{k}) = 1 + 0 - 1 = 0$
   - $(\mathbf{i} + \mathbf{k}) \cdot (-\mathbf{i} + \mathbf{j} + \mathbf{k}) = -1 + 0 + 1 = 0$
   - $(\mathbf{i} + 2\mathbf{j} - \mathbf{k}) \cdot (-\mathbf{i} + \mathbf{j} + \mathbf{k}) = -1 + 2 - 1 = 0$

   Thus, because all of the pairwise dot products are zero, the three vectors are mutually perpendicular.